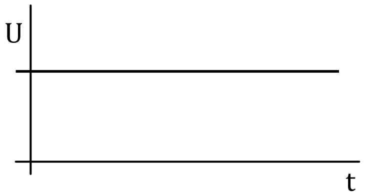
Časově závislé obvody

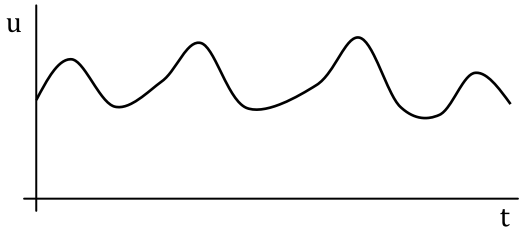
U = konst.

I = konst.

R = konst.



doposud



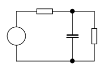
$u = R.i \quad u(t) = R.i(t)$

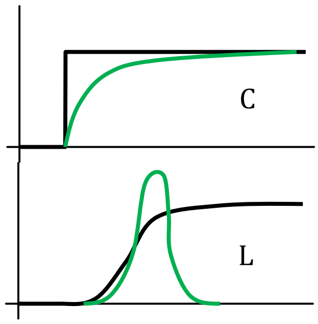
$C = \quad \sum i(t) = 0$

$L = \quad \sum u(t) = 0$

$u = L.\frac{di}{dt} \Rightarrow u(t) = L.\frac{di(t)}{dt}$

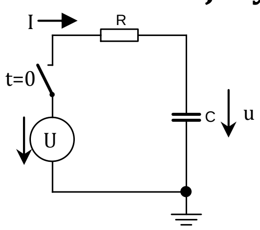
$i = C.\frac{du}{dt} \Rightarrow i(t) = C.\frac{du(t)}{dt}$





- neperiodické obvody – přechodové jevy
- periodické
  - neharmonické – pomocí Fourierovy řady $u(t) = \sum_{n=1}^{\infty} U_n.\sin(\omega_n t + \varphi_n) + U_0$
  - **harmonické** – $u(t) = U_0.\sin(\omega t + \varphi)$

Přechodové jevy



$R.i = U_R = U - u$

$i = \frac{U - u}{R}$

$i = C.\frac{du}{dt} \qquad C = \frac{dq}{du} \Rightarrow C.du = dq; \frac{dq}{dt} = i = C.\frac{du}{dt}$

$\frac{U - u}{R} = C.\frac{du}{dt} \quad |.C$

$\frac{U - u}{R}.C = \frac{du}{dt}$

$\frac{U}{R.C} = \frac{du}{dt} + \frac{u}{R.C}$

$\frac{du}{dt} + \frac{1}{R.C}.u = \frac{U}{R.C}$

$u = \underbrace{u_0}_{K.e^{\lambda t}} + u_p$

$$\lambda.K.e^{\lambda t} + \frac{1}{R.C}.K.e^{\lambda t} = 0$$

$$\lambda + \frac{1}{R.C} = 0$$

$$\lambda = -\frac{1}{R.C} \qquad R.C = \tau$$

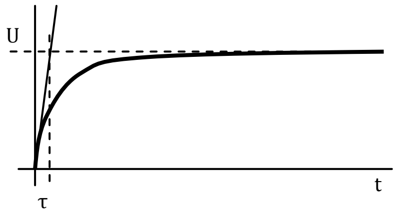
$$u_0 = K.e^{-\frac{t}{R.C}} = K.e^{-\frac{t}{\tau}}$$

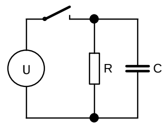
$$u = K.e^{-\frac{t}{\tau}} + L$$

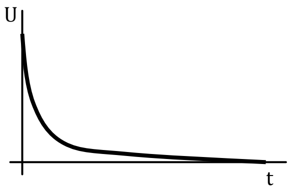
$$t = 0: 0 = K + L \Rightarrow K = -L$$

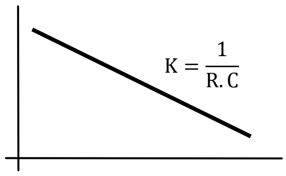
$$t = \infty: U = L$$

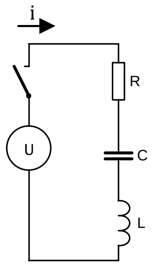
$$u = u_0 + u_p = -U.e^{-\frac{t}{R.C}} + U = U.\left(1 - e^{-\frac{t}{R.C}}\right)$$











$$U = u_R + u_C + u_L$$

$$U = R.i + \int \frac{1}{C}.i.dt + L.\frac{di}{dt} \quad |.\frac{d}{dt}$$

$$0 = R.\frac{di}{dt} + \frac{1}{C}.i + L.\frac{d^2i}{dt^2} \quad |.\frac{1}{L}$$

$$L = \frac{d\Phi}{di} \Rightarrow d\Phi = L.di$$

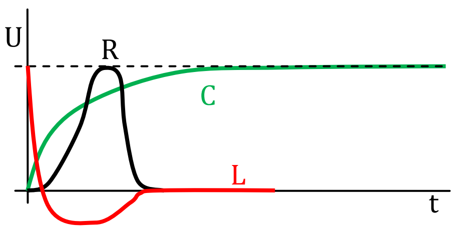
$$u = \frac{d\Phi}{dt} \Rightarrow d\Phi = u.dt$$

$$\Rightarrow L.\frac{di}{dt} = u$$

$$i = C.\frac{du}{dt} \Rightarrow \int \frac{1}{C}.i.dt = u$$

Pro $\lambda_1 \neq \lambda_2 \epsilon R$:

$$i(t) = K_1.e^{\lambda_1 t} + K_2.e^{\lambda_2 t}$$



$$\frac{d^2i}{dt^2} + \frac{R}{L}.\frac{di}{dt} + \frac{1}{L.C}i = 0$$

$$i(t) = i_0 \Rightarrow i(t) = K.e^{\lambda t}$$

$$\lambda^2.K.e^{\lambda t} + \frac{R}{L}.\lambda.K.e^{\lambda t} + \frac{1}{L.C}.K.e^{\lambda t} = 0$$

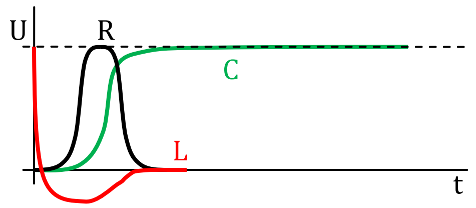
$$\lambda^2 + \frac{R}{L}\lambda + \frac{1}{L.C} = 0$$

$$\lambda_{1,2} = -\frac{R}{2.L} \pm \sqrt{\left(\frac{R}{2.L}\right)^2 - \frac{1}{L.C}}; \quad \frac{R}{2.L} = \beta; \quad \frac{1}{L.C} = \omega_r^2$$

$$\lambda_{1,2} = -\beta \pm \sqrt{\beta^2 - \omega_r^2}$$

$$i(t) = K_1.e^{\lambda_1 t} + K_2.e^{\lambda_2 t}$$

Pro $\lambda_1 = \lambda_2 \epsilon R$:



Pro $\lambda_1 = \overline{\lambda_2} \epsilon C$:

$$\lambda_{1,2} = -\beta \pm j.\sqrt{\omega_r^2 - \beta^2}$$

$$i(t) = K_1.e^{\lambda_1 t} + K_2.e^{\lambda_2.t} = K_1.e^{-\beta t}.e^{j.\sqrt{\omega_r^2-\beta^2}} + K_2.e^{-\beta t}.e^{-j.\sqrt{\omega_r^2-\beta^2}} =$$

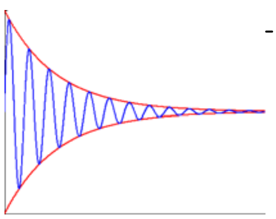
$$= K_1.e^{-\beta t}.\left(\cos\sqrt{\omega_r^2 - \beta^2} + j.\sin\sqrt{\omega_r^2 - \beta^2}\right) +$$

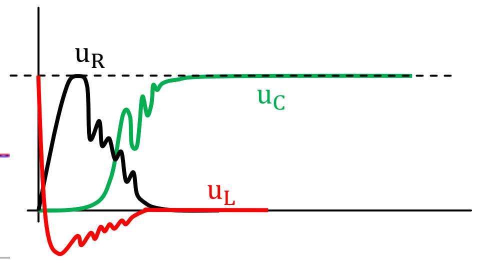
$$+ K_2.e^{-\beta t}.\left(\cos\sqrt{\omega_r^2 - \beta^2} - j.\sin\sqrt{\omega_r^2 - \beta^2}\right) =$$

$$= \underbrace{(K_1 + K_2)}_{C_1}.e^{-\beta t}.\cos\sqrt{\omega_r^2 - \beta^2} + \underbrace{j.(K_1 + K_2)}_{C_2}.e^{-\beta t}.\sin\sqrt{\omega_r^2 - \beta^2}$$

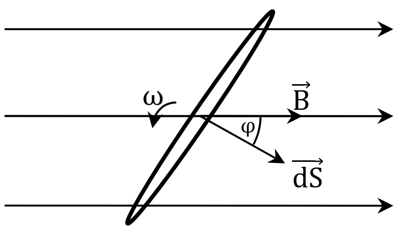
$$i(t) = C_1.e^{-\beta t}.\cos\sqrt{\omega_r^2 - \beta^2} + C_2.e^{-\beta t}.\sin\sqrt{\omega_r^2 - \beta^2}$$

$$i = e^{-\beta t}.\left(C_1.\cos\sqrt{\omega_r^2 - \beta^2} + C_2.\sin\sqrt{\omega_r^2 - \beta^2}\right)$$





Harmonické průběhy



$$u = \frac{d\Phi}{dt} = -\frac{d\iint_S \vec{B}.\vec{dS}}{dt} = -\frac{d\iint_S B.dS.\cos\alpha}{dt} =$$

$$= -\frac{d\left(B.\cos\alpha\iint_S dS\right)}{dt} = -\frac{d(B.S.\cos\alpha)}{dt} =$$

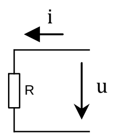
$$= -B.S.\frac{d\cos\alpha}{dt} = B.S.\sin\alpha.\frac{d\alpha}{dt} =$$

$$= \omega.B.S.\sin\alpha = \underbrace{\omega.B.S}_{U_m}.\sin(\omega t + \varphi) =$$

$$= U_m.\sin(\omega t + \varphi)$$

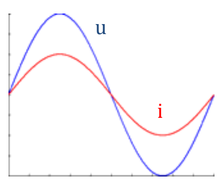
$$\alpha = \alpha(t) = \omega t + \varphi$$

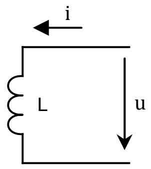
$$\frac{d\alpha}{dt} = \omega$$



$$i = I_m.\sin(\omega t + \varphi)$$

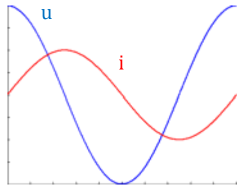
$$u = R.i = \underbrace{R.I_m}_{U_m}.\sin(\omega t + \varphi) = U_m.\sin(\omega t + \varphi)$$

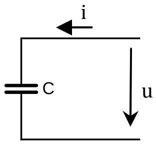




$$i = I_m . \sin(\omega t + \varphi)$$

$$u = L.\frac{di}{dt} = \underbrace{L.\omega.I_m}_{U_m}.\cos(\omega t + \varphi) = U_m.\sin\left(\omega t + \varphi + \frac{\pi}{2}\right)$$

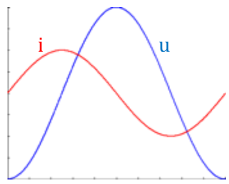




$$i = I_m . \sin(\omega t + \varphi)$$

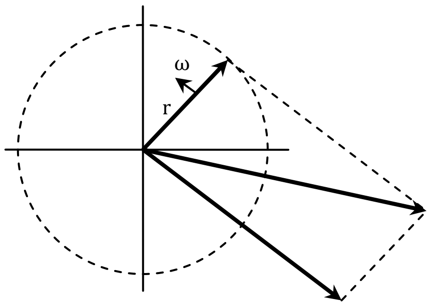
$$u = \frac{1}{C}.\int i.dt = \frac{1}{C}.\int I_m.\sin(\omega t + \varphi).dt = \frac{I_m}{C}.\int \sin(\omega t + \varphi).dt =$$

$$= \frac{I_m}{C.\omega}.[-\cos(\omega t + \varphi)] = \underbrace{\frac{1}{\omega.C}.I_m}_{U_m}.\sin\left(\omega t + \varphi - \frac{\pi}{2}\right)$$



$$\sum i = 0 \quad I_{m_1}.\sin(\omega t + \varphi_1) + \cdots = 0$$

$$\sum u = 0 \quad U_{m_1}.\sin(\omega t + \varphi_1) + \cdots = 0$$



$$r.\sin(\omega t + \varphi) = y$$

$$e^{jx} = \cos x + j.\sin x$$

$$U_m.\sin(\omega t + \varphi)$$

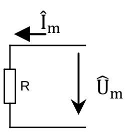
$$\mathrm{Im}\{U_m.e^{jx}\} \epsilon R = U_m.\sin(\omega t + \varphi)$$

$$e^{jx} = e^{j.(\omega t + \varphi)} = \underbrace{e^{j\varphi}}_{\neq f(t)}.e^{j.\omega t}$$

$$u = \mathrm{Im}\left\{\underbrace{U_m.e^{j\varphi}}_{\hat{U}_m}.e^{j\omega t}\right\} = U_m.e^{j.(\omega t + \varphi)}$$

$$\sum u = 0 = \mathrm{Im}\{\hat{U}_{m_1}.e^{j\omega t} + \hat{U}_{m_2}.e^{j\omega t}\}$$

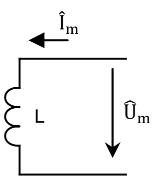
$$\sum \hat{U}_m = 0$$



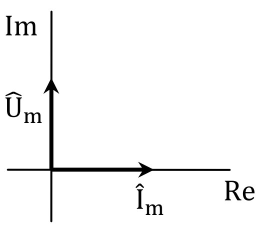
$$u = R.I$$

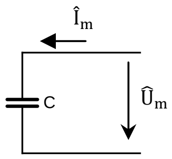
$$\hat{U}_m = R.\hat{I}_m = \mathrm{Im}\left\{\underbrace{R.I_m.e^{j\varphi}}_{\hat{U}_m}.e^{j\omega t}\right\}$$

- proud a napětí jsou ve fázi

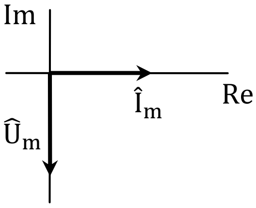


$$u = L.\frac{di}{dt} = \mathrm{Im}\left\{L.\frac{d(I_m.e^{j\varphi}.e^{j\omega t})}{dt}\right\} = \mathrm{Im}\{L.I_m.e^{j\varphi}.e^{j\omega t}.j\omega\} = \mathrm{Im}\left\{\underbrace{j\omega L.I_m.e^{j\varphi}}_{\hat{U}_m}.e^{j\omega t}\right\}$$





$$u = \frac{1}{C}.\int i.\,dt = \mathrm{Im}\left\{\underbrace{\frac{1}{C.\,j\omega}.\,I_m}_{\hat{U}_m}.\,e^{j\omega t}\right\}$$



$\hat{U}_m = R.\,\hat{I}_m$ – odpor

$\hat{U}_m = j\omega L.\,\hat{I}_m$ – cívka

$\hat{U}_m = \frac{1}{j\omega C}.\,\hat{I}_m$ – kondenzátor

$\hat{U}_m = \hat{Z}.\,\hat{I}_m$