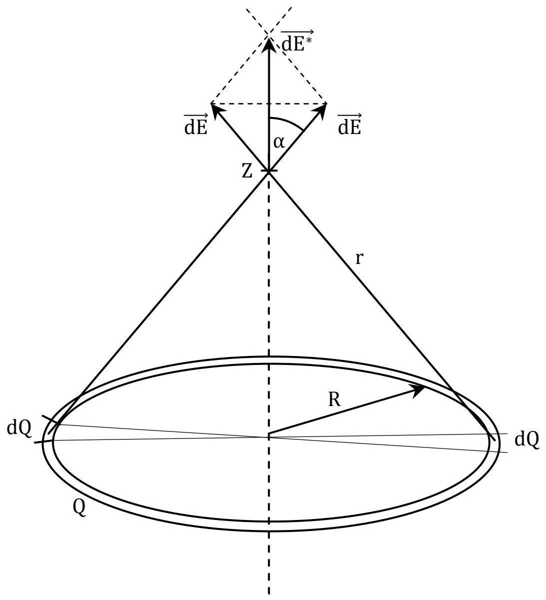
Př. určete $\vec{E}$ =? vodivého prstence: je zadáno R, Q; v místě vzdáleného z ⊥ σ

$$\oiint_S \underbrace{\vec{D}}_{\varepsilon.\vec{E}} . \vec{dS} = Q$$

$$\vec{E} = \frac{1}{4\pi\varepsilon} . \frac{Q}{r^3} . \vec{r}$$

$$\vec{dE} = \frac{1}{4\pi\varepsilon} . \frac{dQ}{r^3} . \vec{r}$$

$$dE = \frac{1}{4\pi\varepsilon} . \frac{dQ}{r^2}$$



$$\vec{dE^*} = 2. dE. \cos\alpha =$$

$$= 2. \frac{1}{4\pi\varepsilon} . \frac{dQ}{r^2} . \cos\alpha =$$

$$= 2. \frac{1}{4\pi\varepsilon} . \frac{dQ}{r^2} . \frac{Z}{r} = 2. \frac{1}{4\pi\varepsilon} . \frac{dQ. Z}{r^3} =$$

$$= \frac{dQ}{2\pi\varepsilon} . \frac{Z}{(R^2 + Z^2)^{3/2}}$$

$$r^2 = R^2 + Z^2$$

$$\cos\alpha = \frac{Z}{r} = \frac{Z}{\sqrt{R^2 + Z^2}}$$

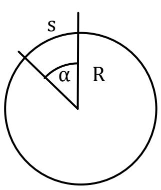
$Q = \tau . l \Rightarrow dQ = \tau . dl$

$\tau = \frac{Q}{l} = \frac{Q}{2\pi R} \Rightarrow dQ = \frac{Q}{2\pi R} . dl$

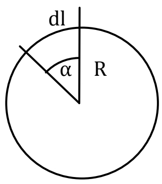
$$dE = \frac{1}{2} \int_O dE^* = \frac{1}{2} \int_Q \frac{Z}{(R^2 + Z^2)^{\frac{3}{2}}} . \frac{dQ}{2\pi\varepsilon} = \frac{1}{2} . \frac{Z}{2\pi\varepsilon(R^2 + Z^2)^{\frac{3}{2}}} . \int_Q dQ = \frac{Z}{4\pi\varepsilon(R^2 + Z^2)^{\frac{3}{2}}} . \int_l \frac{Q}{2\pi R} . dl =$$

$$= \frac{ZQ}{R. 8. \pi^2 \varepsilon (R^2 + Z^2)^{\frac{3}{2}}} . \underbrace{\int_l dl}_{\text{délka}} = \frac{Z. Q. 2\pi R}{8\pi^2 R\varepsilon(R^2 + Z^2)^{\frac{3}{2}}} = \boxed{\frac{Z. Q}{4\pi\varepsilon(R^2 + Z^2)^{\frac{3}{2}}}}$$

$$E = \frac{Z. Q}{4\pi\varepsilon(R^2 + Z^2)^{\frac{3}{2}}} \qquad E = E(z)$$



$$\varphi = \frac{s}{R}$$



$$d\varphi = \frac{dl}{R}$$

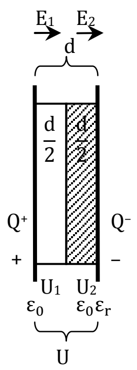
$$dl = d\varphi . R$$

$$\int_l dl = \int_0^{2\pi} R. d\varphi = R. \int_0^{2\pi} d\varphi =$$

$$= R. 2\pi$$

Př. C=?, pole v dielektriku = ?, zadáno U.

1.



$$\oiint \vec{D}. \vec{dS} = Q$$

$$E = \frac{\sigma}{\varepsilon}$$

$$\vec{D} = \varepsilon_0 . \varepsilon_r . \vec{E}$$

$\vec{D_1} = \vec{D_2}$ – pro kolmé složky

$$\varepsilon_0 . \vec{E_1} = \varepsilon_0 . \varepsilon_r . \vec{E_2} \Rightarrow E_2 = \frac{1}{\varepsilon_r} . E_1$$

$$U = U_1 + U_2 = E_1 . \frac{d}{2} + E_2 . \frac{d}{2} = \frac{d}{2} . (E_1 + E_2) =$$

$$= \frac{d}{2} . \left(E_1 + \frac{1}{\varepsilon_r} . E_1\right)$$

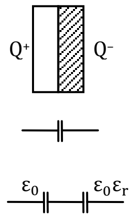
$$U = E. d$$

$$U = E. x$$

$$U=\frac{d}{2}.E_1\left(1+\frac{1}{\varepsilon_r}\right)\Rightarrow E_1=\frac{2.U}{d\left(1+\frac{1}{\varepsilon_r}\right)};\ E_2=\frac{2.U}{d\left(\frac{1}{\varepsilon_r}+\frac{1}{\varepsilon_r^2}\right)}=\frac{2.U.\varepsilon_r}{d\left(1+\frac{1}{\varepsilon_r}\right)}$$

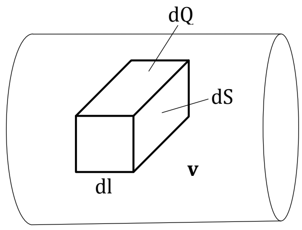
$$C=\frac{Q}{U}=\frac{Q}{\frac{d}{2}\underbrace{E_1}_{\frac{\sigma}{\varepsilon_0}}\left(1+\frac{1}{\varepsilon_r}\right)}=\frac{Q}{\frac{d}{2}.\frac{\sigma}{\varepsilon_0}\left(1+\frac{1}{\varepsilon_r}\right)}=\frac{\sigma.S}{\frac{d}{2}.\frac{\sigma}{\varepsilon_0}\left(1+\frac{1}{\varepsilon_r}\right)}=\frac{2.S.\varepsilon_0}{d\left(1+\frac{1}{\varepsilon_r}\right)}$$

2.



$$C=\frac{C_0.C_1}{C_0+C_1}=\frac{\varepsilon_0.\frac{S}{\frac{d}{2}}.\varepsilon_0.\varepsilon_r.\frac{S}{\frac{d}{2}}}{\varepsilon_0.\frac{S}{\frac{d}{2}}+\varepsilon_0.\varepsilon_r.\frac{S}{\frac{d}{2}}}=\frac{\varepsilon_r.\varepsilon_0.\frac{S}{\frac{d}{2}}}{\frac{1}{\varepsilon_r}+\varepsilon_r}=\frac{2.\varepsilon_0.S}{d.\left(1+\frac{1}{\varepsilon_r}\right)}$$

Jouleův zákon



$$p=\frac{dP}{dV}$$

$$P=\underbrace{\vec{F}}_{E.(-e)}.\vec{v}\qquad \frac{A}{t}=\frac{F.s}{t}=F.v$$

$$P=\vec{F}.\vec{v}$$

$$\vec{j}=\rho.\vec{v}$$

$$p=\frac{dP}{dV}=\frac{d\left(Q.\overbrace{\vec{E}}^{konst.}.\vec{v}\right)}{dV}=\vec{E}.\vec{v}.\frac{dQ}{dV}=\vec{E}.\vec{v}.\rho=\vec{E}.\vec{j}$$

$p=\vec{E}.\vec{j}\ [J.s^{-1}.m^{-3}=W.m^{-3}]$ – diverenciální tvar

$$dP=p.dV$$

$$P=\iiint_V p.dV=\iiint_V \vec{E}.\vec{j}.dV=\iiint_V \vec{E}.\vec{j}.\vec{dl}.\vec{dS}=\iiint_V \vec{E}.\vec{dl}.\vec{j}.\vec{dS}=\underbrace{\int_l \vec{E}.\vec{dl}}_{U}.\underbrace{\iint_S \vec{j}.\vec{dS}}_{I}$$

$P=U.I\ [J.s^{-1}=W]$ – integrální tvar

$$U=R.I\Rightarrow P=R.I^2$$

$$I=\frac{U}{R}\Rightarrow P=\frac{U^2}{R}$$

Energie

$$A = \int_{t_1}^{t_2} P.\,dt \qquad P = \text{konst.} \Rightarrow P.\int dt = P.\,t[J]$$

Proud kondukční, konvekční, posuvný

Kondukční

$\vec{j} = \sigma.\vec{E} -$ pohyb elektronů uvnitř vodiče.

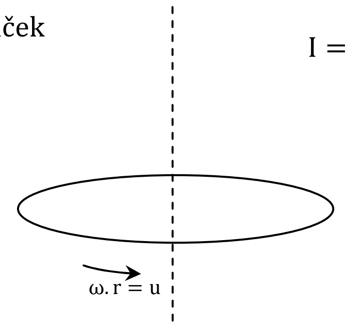
$\vec{j} = \vec{v}$

Konvekční

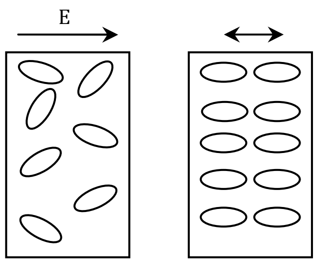
např. otáčivý kotouček

$\vec{j} = \vec{u}.\,\rho$

$$I = \frac{dQ}{dt}$$



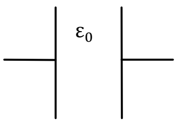
Posuvný



$$I_P = \frac{dQ}{dt} \qquad \oiint \vec{D}.\vec{dS} = Q$$

$$I_P = \frac{\partial \iint \vec{D}.\vec{dS}}{\partial t} = \iint \underbrace{\frac{\partial \vec{D}}{\partial t}}_{\vec{j_P}}.\vec{dS}$$

$$\vec{j_P} = \frac{\partial \vec{D}}{\partial t}$$



$$\vec{j_P} = \varepsilon_0.\frac{\partial \vec{E}}{\partial t} + \frac{\partial \vec{P}}{\partial t}$$

$$\vec{D} = \varepsilon_0.\varepsilon_r.\vec{E} = \varepsilon_0.\vec{E} + \vec{P}$$

Maxvelovy rovnice

1.
$$\oiint \vec{D}.\overrightarrow{dS} = Q \quad div\vec{D} = \rho$$

2.
$$\oiint \vec{B}.\overrightarrow{dS} = 0 \quad div\vec{B} = 0 \rightarrow \text{žádný magnetický náboj}$$

3.
$$\oint \vec{E}.\overrightarrow{dl} = \iint -\frac{\partial\vec{B}}{\partial t}.\overrightarrow{dS} - \underbrace{}_{}\overset{\sum U = 0}{} \text{ napětí vzniká změnou mag. pole} \quad rot\vec{E} = -\frac{\partial B}{\partial t}$$

4.
$$\oint \vec{H}.\overrightarrow{dl} = \iint \left(\vec{j} + \frac{\partial\vec{D}}{\partial t}\right).\overset{\sum I = 0}{\overrightarrow{dS}} \quad rot\vec{H} = \vec{j} + \frac{\partial\vec{D}}{\partial t}$$