$D = \varepsilon_0 \varepsilon_r E$

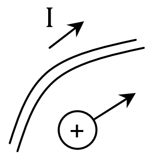
Elektrokinetika

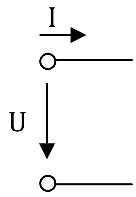
Elektrický proud

- uspořádaný pohyb nábojů, u vodičů elektrony, polovodiče oba.

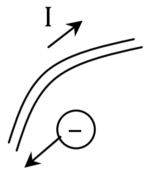
$$\boxed{I = \frac{dQ}{dt}}$$

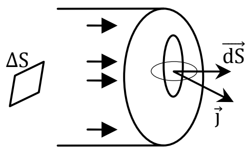
I kladný, tak jak kladný náboj





I záporný





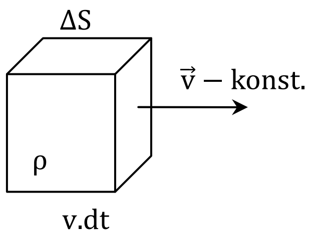
$\vec{j}$ − hustota proudu

$$I = \iint_S \vec{j} . \vec{dS}$$

$$\frac{I}{dS.\cos\alpha} = j [A.m^{-2}]$$

$$\frac{I}{S} = j \qquad I = \iint_S j . dS.\cos\alpha$$

$$I = \iint_S \vec{j} . \vec{dS}$$



$$dQ = \rho . dV = \rho . \Delta S . v . dt$$

$$I = \frac{dQ}{dt}$$

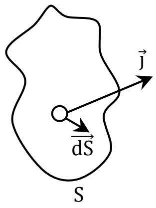
$$dQ = I . dt = j . \Delta S . dt$$

$$\rho . \Delta S . v . dt = j . \Delta S . dt$$

$$\vec{j} = \rho . \vec{v}$$

$$\oiint_S \vec{f} . \vec{dS} = \iiint_V \operatorname{div}\vec{f} . dV$$

Rovnice kontinuity



$$I = \oiint_S \vec{j}.\overrightarrow{dS} = -\frac{\partial Q}{\partial t} = -\frac{\partial}{\partial t}\iiint_V \rho.\,dV$$

$$Q = \iiint_V \rho.\,dV$$

$$\rho(x;y;z;t)$$

$$\iiint_V \mathrm{div}\vec{j}.\,dV = -\frac{\partial}{\partial t}\iiint_V \rho.\,dV \Rightarrow \mathrm{div}\vec{j} = -\frac{\partial \rho}{\partial t}$$

$$\boxed{\frac{\partial \rho}{\partial t} + \mathrm{div}\vec{j} = 0}$$

$\frac{\partial Q}{\partial t} = 0$ – náboj nevzniká ani nezaniká

$$Q = \rho.V \quad \rho = \rho(x;y;z;t)$$
$$x = x(t)$$
$$y = y(t)$$
$$z = z(t)$$

$V = \text{konst.}$

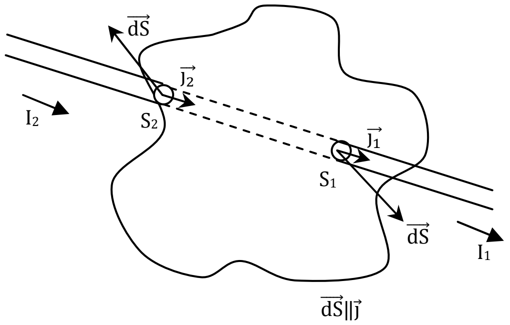
$$\frac{d(\rho.V)}{dt} = 0$$

$$\frac{d\rho}{dt} = 0 = \frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial x}.\overbrace{\frac{\partial x}{\partial t}}^{v_x} + \frac{\partial \rho}{\partial y}.\overbrace{\frac{\partial y}{\partial t}}^{v_y} + \frac{\partial \rho}{\partial z}.\overbrace{\frac{\partial z}{\partial t}}^{v_z}$$

$$\frac{d\rho}{dt} = 0 = \frac{\partial \rho}{\partial t} + \left(\frac{\partial \rho}{\partial x};\frac{\partial \rho}{\partial y};\frac{\partial \rho}{\partial z}\right).\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \nabla\rho.\vec{v} + \frac{\partial \rho}{\partial t} = \nabla(\rho.\vec{v}) = \mathrm{div}(\rho.\vec{v}) = \mathrm{div}\vec{j}$$

$$0 = \frac{\partial \rho}{\partial t} + \mathrm{div}\vec{j}$$

I. Kirchhoffův zákon

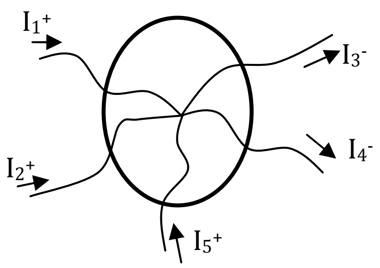


$$I = \oint\!\!\!\oint_S \vec{j}.\overrightarrow{dS} = \iint_{S_1} \vec{j_1}.dS - \iint_{S_2} \vec{j_2}.dS = 0 = I_1 - I_2 \Rightarrow$$

$$\Rightarrow I_1 = I_2$$

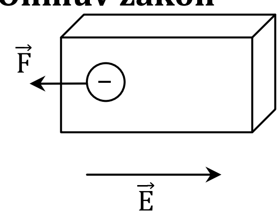
$$\mathrm{div}\vec{j} = 0$$

$$\oint\!\!\!\oint \vec{j}.\overrightarrow{dS} = 0$$



$$\sum_{i=1}^{4} I_i = 0 \qquad \boxed{\sum_i I_i = 0}$$

Ohmův zákon

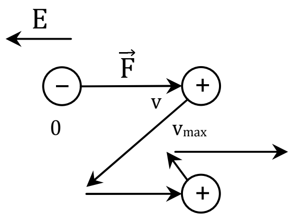


$$F = -e.\vec{E} = m.\vec{a} = m.\frac{d\vec{v}}{dt}$$

$$-e.\vec{E} = m.\frac{d\vec{v}}{dt}$$

$$\frac{-e.\vec{E}}{m}.dt = d\vec{v} \quad \int$$

$$\frac{-e.\vec{E}}{m}.t = \vec{v}$$



(relaxační doba)

$$-\frac{1}{2}.\frac{e.\vec{E}}{m}.\bar{t} = \bar{v}$$

$$\vec{v} = \frac{0 + v_{max}}{2}$$

$$\vec{j} = \rho.\vec{v}$$

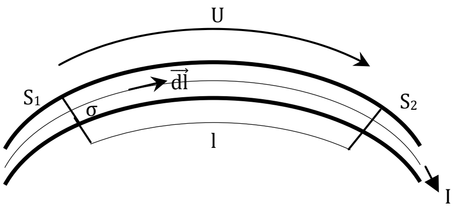
koncentrace $\frac{N}{V}$

$$\rho = \frac{Q}{V} = \frac{-eN}{V} = -e.n\ [C.m^{-3}]$$

$$\vec{j} = e.n.\frac{e.\vec{E}}{2.n}.\bar{t} = \underbrace{\frac{n.e^2.\bar{t}}{2.m}}_{\sigma}.\vec{E}$$

měrná vodivost $[\Omega^{-1}.m^{-1} = S.m^{-1}]$

$\boxed{\vec{j} = \sigma . \vec{E}}$ **Ohmův zákon** v diferenciálním tvaru



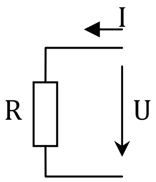
$$\rho = \frac{1}{\sigma} - \text{ měrný odpor } [\Omega . \mathrm{m}]$$

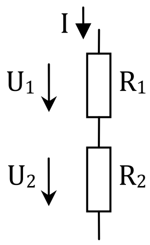
$$U = \int_l \vec{E} . \vec{dl} = \int_l E . dl = \int_l \frac{j}{\sigma} dl = \int_l \frac{I}{S . \sigma} dl = I . \int_l \frac{dl}{S . \sigma} =$$

$$\vec{E} \| \vec{dl} \qquad = I \underbrace{\int_l \rho . \frac{dl}{S}}_{R}$$

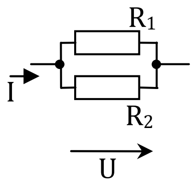
$\boxed{U = I . R}$ Ohmův zákon

$$R = \frac{U}{I} \; [\Omega] \qquad G = \frac{I}{U} \; [S]$$





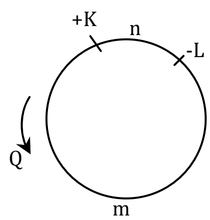
$$U = U_1 + U_2$$
$$R . I = R_1 . I + R_2 . I$$
$$R = R_1 + R_2$$
$$R = \sum_i R_i$$

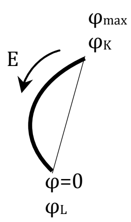


$$-I + I_1 + I_2 = 0$$
$$I_1 + I_2 = I$$
$$\frac{U}{R} = \frac{U}{R_1} + \frac{U}{R_2}$$
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R} = \sum_i R_i$$

Elektromotorické napětí





$$\vec{j} = \sigma . \vec{E}$$

$$U_{KL} = \varphi_K - \varphi_L$$

$$m: U_{KL} = \int_K^L \vec{E_S} . \vec{dr} \qquad \vec{E} = \vec{E_s} \qquad I = \frac{U_{KL}}{R} = \frac{1}{R} . \int_K^L \vec{E_S} . \vec{dr}$$

$$n: U_{LK} = \int_L^K (\vec{E_S} + \vec{E^*}).\vec{dr} \qquad \vec{E} = \vec{E_S} + \vec{E^*}$$

$$\oint \vec{E}.\vec{dr} = \int_K^L \vec{E_S}.\vec{dr} + \int_L^K (\vec{E_S} + \vec{E^*}).\vec{dr} = \underbrace{\int_K^L \vec{E_S}.\vec{dr} + \int_L^K \vec{E_S}.\vec{dr}}_{0} + \int_L^K \vec{E^*}.\vec{dr}$$

$$\oint \vec{E}.\vec{dr} = \boxed{\int_L^K \vec{E^*}.\vec{dr} = U_e}$$ **elektromotorické napětí**

$$n: I = \frac{1}{R_i}\int_L^K (\vec{E_S} + \vec{E^*}).\vec{dr} = \frac{1}{R_i}\underbrace{\int_L^K \vec{E_S}.\vec{dr}}_{-U_{KL}} + \frac{1}{R_i}\underbrace{\int_L^K \vec{E^*}.\vec{dr}}_{U_e} = \frac{-U_{KL}}{R_i} + \frac{U_e}{R_i} = -\frac{R.I}{R_i} + \frac{U_e}{R_i}$$

vnitřní odpor zdroje (R_i)

$$I = -\frac{R.I}{R_i} + \frac{U_e}{R_i} \Rightarrow R_i.I = U_0 - R.I \Rightarrow U_e = (R + R_i).I$$ Ohmův zákon pro uzavřený obvod

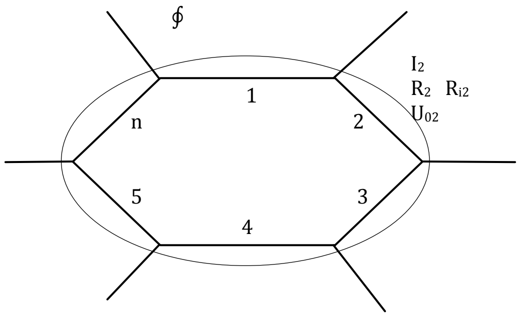
$$U_e = U_S + R_i.I$$

svorkové napětí (U_S)

$R_i = 0$ ideální zdroj napětí

II. Kirchhoffův zákon

$$\oint \vec{E}.\vec{dr} = \underbrace{\oint \vec{E_S}.\vec{dr}}_{0} + \oint \vec{E^*}.\vec{dr} \Rightarrow \int_L^K E^*.dr$$



$$\oint \vec{E}.\vec{dr} = \sum_i^n \int_{m_i} \qquad + \sum_i \int_{n_i}$$

$$\oint \vec{E}.\vec{dr} = \sum_i^k \left[\int_{m_k} \vec{E_S}.\vec{dr} + \int_{n_k} (\vec{E_S} + \vec{E^*}).\vec{dr}\right] =$$

$$= \sum_{i=1}^k R_K.I_K + \sum_{i=1}^k R_{i_k}.I_k$$

$$\oint \vec{E}.\vec{dr} = \sum_{i=1}^k U_{e_i}$$

$$\sum_i U_{e_i} = \sum_i (R_k + R_{i_k}).I_k$$

$$\boxed{\begin{gathered} U = R.I \\ \sum I = 0 \\ \sum U = 0 \end{gathered}}$$

metoda smyčkových proudů

$$\sum_k U_{e_k} = \sum_k R.I_k = \sum U_{e_i} - \sum U_k = 0$$

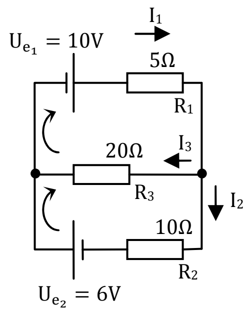
Metoda smyčkových proudů

1. směr
2. proudy + směr proudu
3. U_e + $\longrightarrow$ ⊣⊢
4. úbytek R.I $+ \rightrightarrows$ $- \rightleftarrows$ znaménko podle směru

$$U_{e_1} - U_{e_2} + R_2.I_5 - R_4.I_8 = 0$$

$$\sum I = 0$$

<u>Př.</u>



I. K. Z.

$$I_1 - I_2 - I_3 = 0$$

$$I_2 + I_3 - I_1 = 0$$

III. K. Z.

$$R_1.I_1 + R_3.I_3 - U_{e_1} = 0$$

$$-R_3.I_3 + R_2.I_2 - U_{e_2} = 0$$

$$\left(\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 5 & 0 & 20 & 10 \\ 0 & 10 & -20 & 6 \end{array}\right) \left(\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 1 & 0 & 4 & 2 \\ 0 & 5 & -10 & 3 \end{array}\right) \left(\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 0 & 5 & -10 & 3 \\ 1 & 0 & 4 & 2 \end{array}\right) \left(\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 0 & 5 & -10 & 3 \\ 0 & 1 & 5 & 2 \end{array}\right) \left(\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 0 & 5 & -10 & 3 \\ 0 & 0 & -35 & -7 \end{array}\right)$$

$$I_3 = \frac{7}{35} = \frac{1}{5}A$$

$$5I_2 - 2 = 3 \Rightarrow I_2 = 1A$$

$$I_1 - 1 - \frac{1}{5} = 0 \Rightarrow I_1 = 1 + \frac{1}{5} = \frac{6}{5}A$$