$$A = \int_K^L \vec{F}.\overrightarrow{dr} = \cdots = \frac{Q_0 Q}{4\pi\varepsilon_0}\left(\frac{1}{r_k} - \frac{1}{r_L}\right)$$

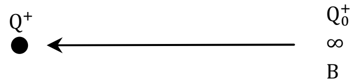
$A = \oint \vec{F}.\overrightarrow{dl} = 0$ **konzervativní pole** (zákon zachov. energie)

→ Lze přiřadit potenciální energii

Potenciální energie

$W_k = A = -\int_B^K \vec{F}.\overrightarrow{dl}$ – práce proti silám z potenciální energie = 0 do bodu K

(B) → Potenciální energie = 0



Normálně se odpuzují → překonáme odpudivou sílu

$$-\int_B^K Q_0\vec{E}.\overrightarrow{dl} = -Q_0\int_B^K \vec{E}.\overrightarrow{dl}$$

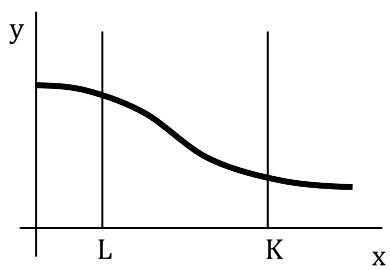
Potenciál

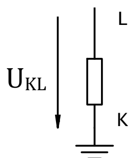
$\varphi = \frac{W_p}{Q_0} = -\int_B^K \vec{E}.\overrightarrow{dl} = \left[\frac{J}{C}\right] = [V]$ skalární veličina

Napětí

$$\varphi_K - \varphi_L = -\int_B^K \vec{E}.\overrightarrow{dl} - \int_B^L \vec{E}.\overrightarrow{dl} = \int_B^L \vec{E}.\overrightarrow{dl} - \int_B^K \vec{E}.\overrightarrow{dl} = -\int_L^K \vec{E}.\overrightarrow{dl}$$

$$\boxed{U_{LK} = \varphi_L - \varphi_K = -\int_L^K \vec{E}.\overrightarrow{dl}}$$

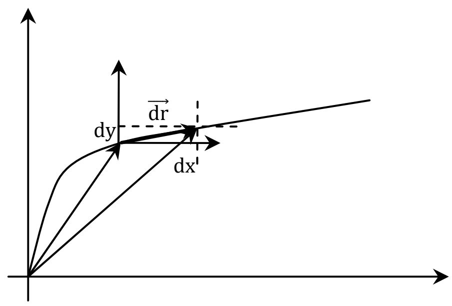




Více nábojů

$$\varphi = -\int_B^K \sum_i \vec{E_i}.\overrightarrow{dl} = -\int_B^K \vec{E_1}.\overrightarrow{dl} - \int_B^K \vec{E_2}.\overrightarrow{dl} - \cdots = \sum_i \underbrace{-\int_B^K \vec{E_i}.\overrightarrow{dl}}_{\varphi_i} = \sum_i \varphi_i$$

$$\boxed{\varphi^{+} = \frac{1}{4\pi\varepsilon}.\frac{Q^{+}}{r}}$$ $$\boxed{\varphi^{-} = \frac{1}{4\pi\varepsilon}.\frac{Q^{-}}{r}}$$



$$\varphi = -\int \vec{E}.\vec{dl} = \varphi(x;y;z)$$

$$d\varphi = -\vec{E}.\vec{dr} = \overrightarrow{\text{grad}\,\varphi}.\vec{dr}$$

$$\vec{E} = E_x.\vec{i} + E_y.\vec{j} + E_z.\vec{k}$$

$$\vec{dr} = dx.\vec{i} + dy.\vec{j} + dz.\vec{k}$$

$$d\varphi = \frac{\partial\varphi}{\partial x}dx + \frac{\partial\varphi}{\partial y}dy + \frac{\partial\varphi}{\partial z}dz$$

$$\frac{\partial\varphi}{\partial x}dx + \frac{\partial\varphi}{\partial y}dy + \frac{\partial\varphi}{\partial z}dz = -\left(E_x.\vec{i} + E_y.\vec{j} + E_z.\vec{k}\right).\left(dx.\vec{i} + dy.\vec{j} + dz.\vec{k}\right)$$

$$\frac{\partial\varphi}{\partial x}dx + \frac{\partial\varphi}{\partial y}dy + \frac{\partial\varphi}{\partial z}dz = -\left(E_x.dx + E_y dy + E_z dz\right)$$

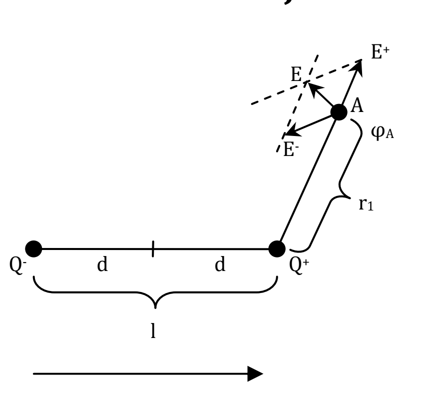
$$\left(\frac{\partial\varphi}{\partial x};\frac{\partial\varphi}{\partial y};\frac{\partial\varphi}{\partial z}\right).\begin{pmatrix}dx\\dy\\dz\end{pmatrix} = \left(-E_x;-E_y;-E_z\right).\begin{pmatrix}dx\\dy\\dz\end{pmatrix}$$

$$\frac{\partial\varphi}{\partial x} = -E_x$$
$$\frac{\partial\varphi}{\partial y} = -E_y \qquad \left(\frac{\partial}{\partial x};\frac{\partial}{\partial y};\frac{\partial}{\partial z}\right)\varphi = -\vec{E}$$
$$\frac{\partial\varphi}{\partial z} = -E_z$$
$$\overrightarrow{\text{grad}\varphi} = -\vec{E}$$
$$\nabla.\varphi = -\vec{E}$$

$$E = -\text{grad}\,\varphi$$

$0 = d\varphi = \overrightarrow{\text{grad}\varphi}.\vec{dr}$ – ekvipotentní energie kolmá na $\vec{dr}$
– největší přírůstek ukazuje $\overrightarrow{\text{grad}}\varphi$

Dva bodové náboje



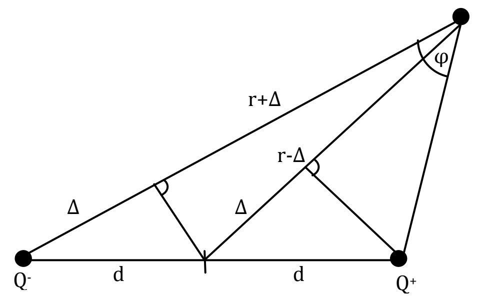
dipól

$$\varphi_A = \varphi_A^+ + \varphi_A^- = \frac{Q}{4\pi\varepsilon}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\varphi_A^+ = \frac{1}{4\pi\varepsilon}.\frac{Q}{r_1}$$

$$\varphi_A^- = \frac{1}{4\pi\varepsilon}.\frac{Q}{r_2}$$

Vzdálenost A dostatečně veliká



$\Delta = d.\cos\varphi$

$r_1 = r - d.\cos\varphi$

$r_2 = r + d.\cos\varphi$

$$\varphi_A = \frac{Q}{4\pi\varepsilon}\left(\frac{1}{r - \cos\varphi} - \frac{1}{r + \cos\varphi}\right)$$

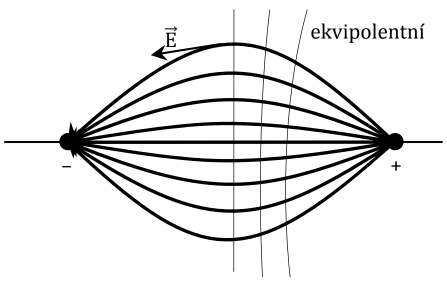
$$\varphi_A = \frac{1}{4\pi\varepsilon}.\frac{Q}{r}\left(\frac{1}{1 - \frac{d.\cos\varphi}{r}} - \frac{1}{1 - \frac{d.\cos\varphi}{r}}\right) \quad d \ll r$$

$$(1 \pm \alpha) = 1 \mp \alpha$$

$$\varphi_A = \frac{1}{4\pi\varepsilon}.\frac{Q}{r}\left(1 + \frac{d.\cos\varphi}{r} - 1 + \frac{d.\cos\varphi}{r}\right) = \frac{1}{4\pi\varepsilon}.\frac{Q}{r^2}.\underbrace{2d}_{l}.\cos\varphi = \frac{\vec{p}.\vec{r_0}}{4\pi\varepsilon r^2}$$

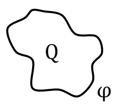
$$Q.\vec{l} = \vec{p}$$

$$\varphi_A = \frac{\vec{p}.\vec{r_0}}{4\pi\varepsilon r^2}$$



Kapacita

$$\varphi = \frac{1}{4\pi\varepsilon}.\frac{Q}{r}$$

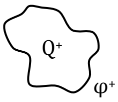


$$\varphi = \frac{1}{4\pi\varepsilon}.\frac{\vec{p}.\vec{r_0}}{r^2} = \frac{Ql}{4\pi\varepsilon r^2}$$

$\varphi \sim Q$ φ úměrný Q

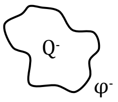
$$C = \frac{Q}{\varphi}\ [F] - \text{ charakterizuje geometrické uspořádání}$$

Kondenzátor

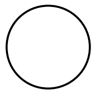


$$C = \frac{Q}{U}$$

$$U = \varphi^{+} - \varphi^{-}$$

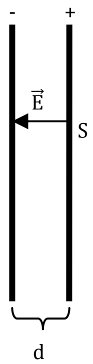


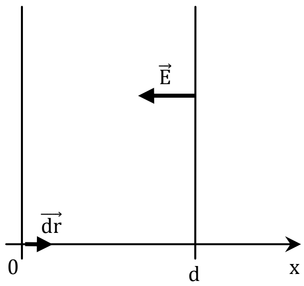
koule



$$\varphi = \frac{1}{4\pi\varepsilon} . \frac{Q}{r} \qquad C = \frac{Q}{\frac{1}{4\pi\varepsilon} . \frac{Q}{r}} = 4\pi\varepsilon r$$

desky

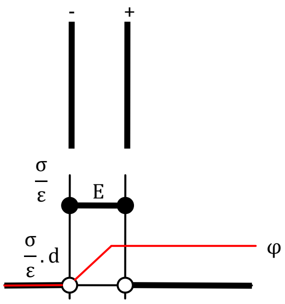




$$E = \frac{\sigma}{\varepsilon}$$

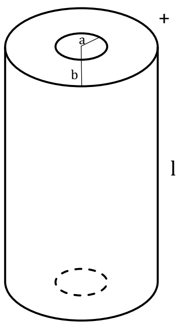
$$U = -\int_0^d \vec{E} . \overrightarrow{dx} = \int_0^d E . dx = \frac{\sigma}{\varepsilon} \int_0^d dx = \frac{\sigma}{\varepsilon} . d$$

$$\varphi = -\int \vec{E}\, dx = \frac{\sigma}{\varepsilon} . x$$



$$C = \frac{Q}{U} = \frac{Q}{\frac{\sigma}{\varepsilon} . d} = \frac{\sigma S}{\frac{\sigma}{\varepsilon} . d} = \boxed{\varepsilon . \frac{S}{d}}$$

válec

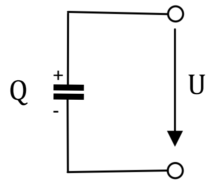


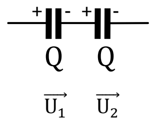
$$E = \frac{1}{4\pi\varepsilon} . \frac{Q}{r . l}$$

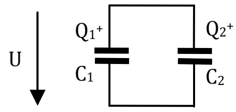
$$U = -\int_a^b \vec{E} . \vec{dl} = \int_a^b \frac{1}{2\pi\varepsilon} . \frac{Q}{r . l} = \frac{Q}{2\pi\varepsilon l} . \int_a^b \frac{dr}{r} = \frac{Q}{2\pi\varepsilon l} \ln \frac{b}{a}$$

$$C = \frac{Q}{\frac{Q}{2\pi\varepsilon l} \ln \frac{b}{a}} = \frac{2\pi\varepsilon l}{\ln \frac{b}{a}}$$

Řazení kondenzátorů







$$Q = Q_1 + Q_2$$

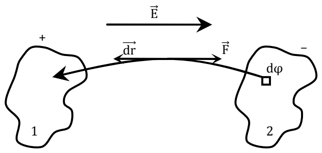
$$C.U = C_1.U + C_2.U$$

$$C = C_1 + C_2$$

$$U = U_1 + U_2$$

$$C = \frac{Q}{U} \Rightarrow \frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} \Rightarrow \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

Energie kondenzátoru



$$dF = dQ.E$$

$$dA = \int_2^1 \overrightarrow{dF}.\overrightarrow{dr} - \int_2^1 dQ.E.dr = -dQ\underbrace{\int_2^1 E.dr}_{U} = dQ.U$$

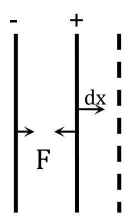
$$A = \int_0^Q dQ.U = \int_0^Q dQ.\frac{Q}{C} = \frac{1}{C}\int_0^Q Q.dQ = \boxed{\frac{1}{2}.\frac{Q^2}{C}} = \frac{1}{2}.\frac{C.U^2}{C} = \boxed{\frac{1}{2}.C.U^2} = \boxed{\frac{1}{2}Q.U}$$

Hustota energie

$$W = \frac{dW}{dV} = \frac{W}{V} = \frac{\frac{1}{2}CU^2}{Sd} = \frac{\frac{1}{2}\varepsilon\frac{S}{d}E^2d^2}{Sd} = \frac{1}{2}\varepsilon E^2 = \frac{1}{2}\vec{D}.\vec{E}$$

$$\varepsilon E = D$$

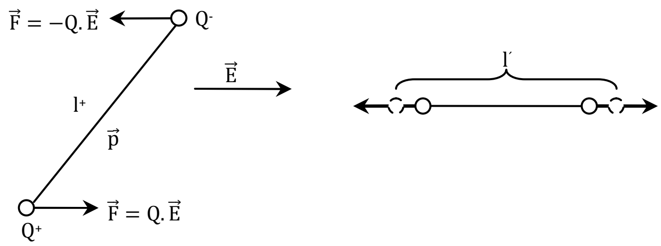
Síla mezi deskami



$$F.dx = W.S.dx = \frac{1}{2}D.E.S.dx$$

$$F = \frac{1}{2}D.E.S = \frac{1}{2}\varepsilon E^2 S = \frac{1}{2}\varepsilon\frac{\sigma^2}{\varepsilon^2}S = \frac{1}{2}\frac{\sigma^2}{\varepsilon}S$$

Dipól v elektrickém poli

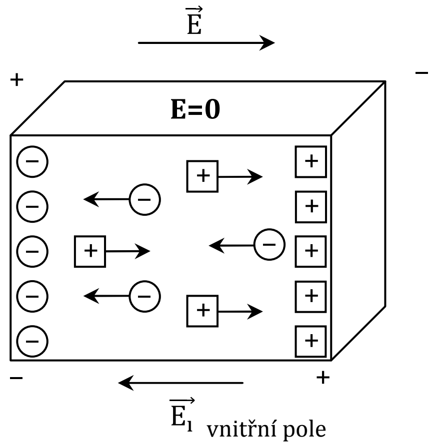


- volně umístěný → otočí se!

$$\vec{M} = \vec{l} \times \vec{F} = \vec{l} \times Q\vec{E} = \underbrace{Q\vec{l}}_{\vec{p}} \times \vec{E}$$

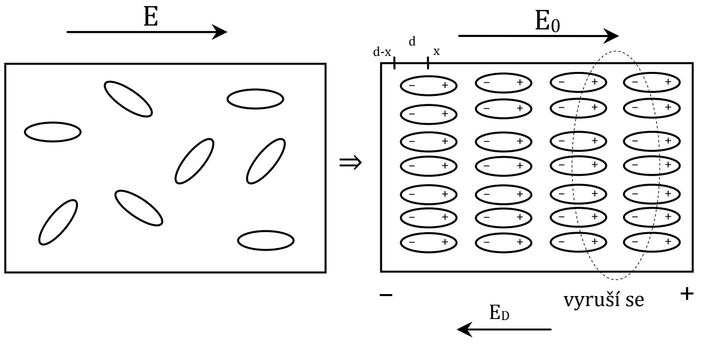
$$\vec{M} = \vec{p} \times \vec{E}$$

Vodič v elektrickém poli



$$E_i = E$$

Izolant v elektrickém poli



$$E = E_0 - E_d$$

$+ \quad n.Q.x.dS$ $\quad -$ kolik projde plochou

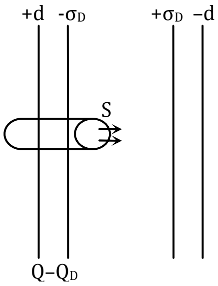
$- \quad n.Q.(d-x).dS$

$$dQ_0 = n.Q.x.dS - n.Q.x.dS + \\ + n.\underbrace{Q.d}_{p}.dS = \vec{p}.dS$$

$$\vec{p} = \frac{\sum \vec{p}}{V} \qquad p = \frac{dQ_D}{dS} = \sigma_D$$

$$\vec{D} = \underbrace{\varepsilon}_{\varepsilon_0 \varepsilon_r} .\vec{E} = \varepsilon_0.\vec{E} + \vec{p}$$

Zobecnění Gaussovy věty



$$\oint\!\!\oint E.\,dS = \frac{Q}{\varepsilon}$$

$$\sigma_D = p$$

$$Q_D = \iint_S \vec{p}.\overrightarrow{dS} = \oint\!\!\oint \vec{p}.\overrightarrow{dS}$$

$$\oint\!\!\oint \vec{D}.\overrightarrow{dS} = Q \Rightarrow \oint\!\!\oint \vec{E}.\overrightarrow{dS} = \frac{Q - Q_D}{\varepsilon_0}$$

$$\iint_P \vec{p}.\overrightarrow{dS} = 0$$

$$\oint\!\!\oint \vec{E}.\overrightarrow{dS} = \frac{Q - \oint\!\!\oint \vec{p}.\overrightarrow{dS}}{\varepsilon_0}$$

$$\oint\!\!\oint \varepsilon_0\vec{E}.\overrightarrow{dS} = Q - \oint\!\!\oint \vec{p}.\overrightarrow{dS}$$

$$\oint\!\!\oint (\varepsilon_0\vec{E} + \vec{p})\overrightarrow{dS} = Q$$

$$\varepsilon_0\vec{E} + \vec{p} = \vec{D}$$