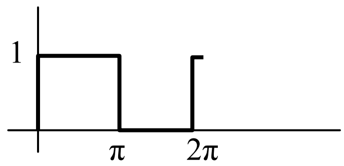
<u>Příklad č. 1</u>

Vypočtěte jednotlivé harmonické složky zadaného signálu pro $n \in \langle 0; 20\rangle$.



$\omega = 1, T = 2\pi$

Výpočet:

$s(t) = 1$ pro $t \in \langle 0; \pi\rangle$

$$c_0 = \frac{1}{T}.\int_0^T s(t).dt = \frac{1}{T}.\int_0^\pi 1.dt = \frac{1}{T}.[t]_0^\pi = \frac{1}{2\pi}.(\pi - 0) = \frac{1}{2}$$

$$c_n = \frac{1}{T}.\int_0^T s(t).e^{-j.n.\omega.t}.dt = \frac{1}{T}.\int_0^\pi e^{-j.n.\omega.t}.dt = \frac{1}{T}.\left[\frac{e^{-j.n.\omega.t}}{-j.n.\omega}\right]_0^\pi =$$

$$= \frac{1}{T}.\left[\frac{e^{-j.n.\omega.\pi} - e^{-j.n.\omega.0}}{-j.n.\omega}\right] = \frac{1}{T}.\left\{\frac{\cos(-n.\omega.\pi) + j.\sin(-n.\omega.\pi) - 1}{-j.n.\omega}\right\} =$$

$$= \frac{1}{T}.\left\{\frac{\cos(n.\omega.\pi) - j.\sin(n.\omega.\pi) - 1}{-j.n.\omega}\right\} =$$

$$= \frac{1}{2\pi}\left\{\frac{\cos(n.\omega.\pi) - j.\sin(n.\omega.\pi) - 1}{-j.n.\omega}\right\}$$

$$c_n = \frac{1}{2}.a_n - j.\frac{1}{2}.b_n$$

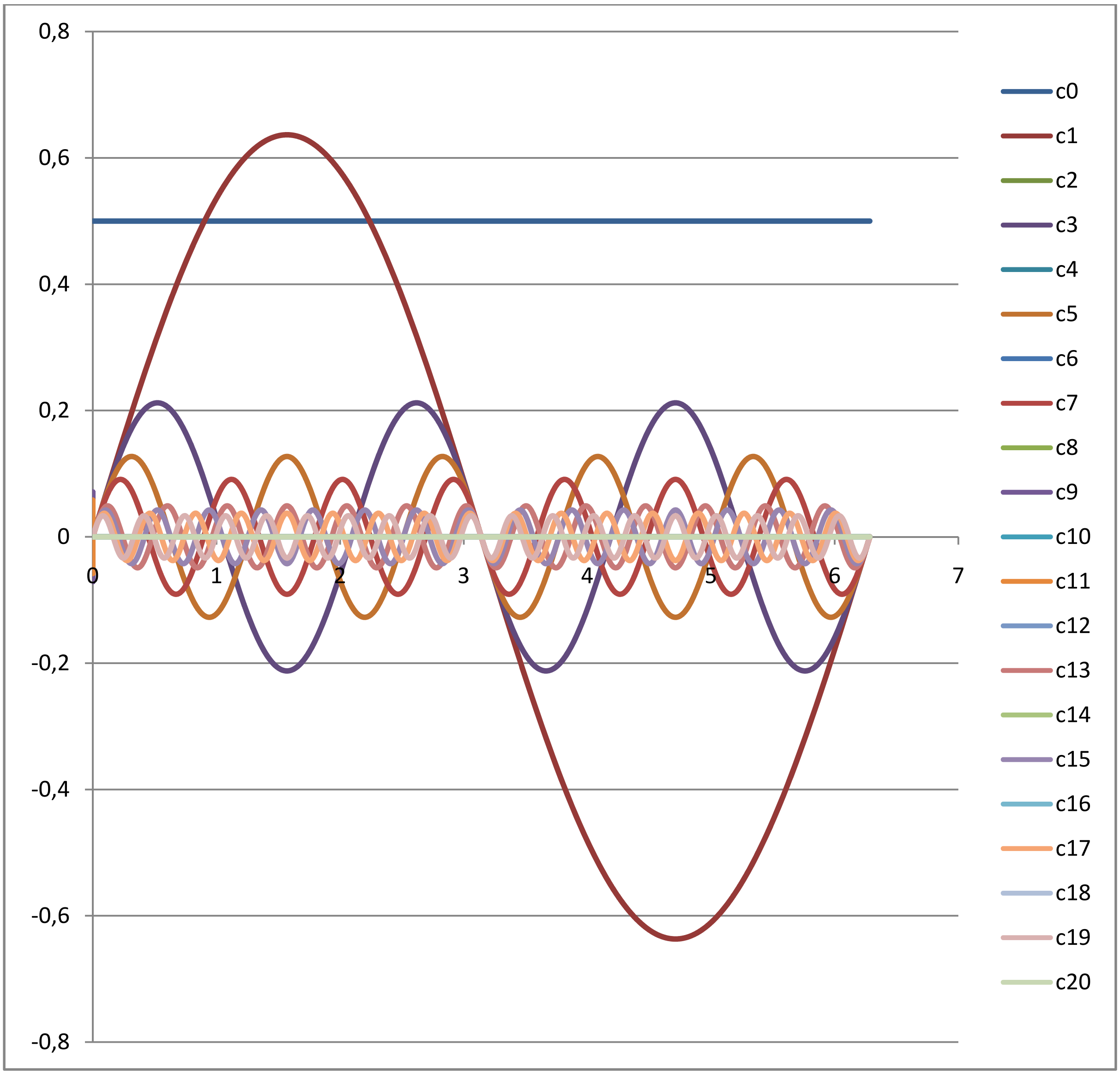
$$a_n = 2.\mathrm{Re}(c_n)$$

$$b_n = -2.\mathrm{Im}(c_n)$$

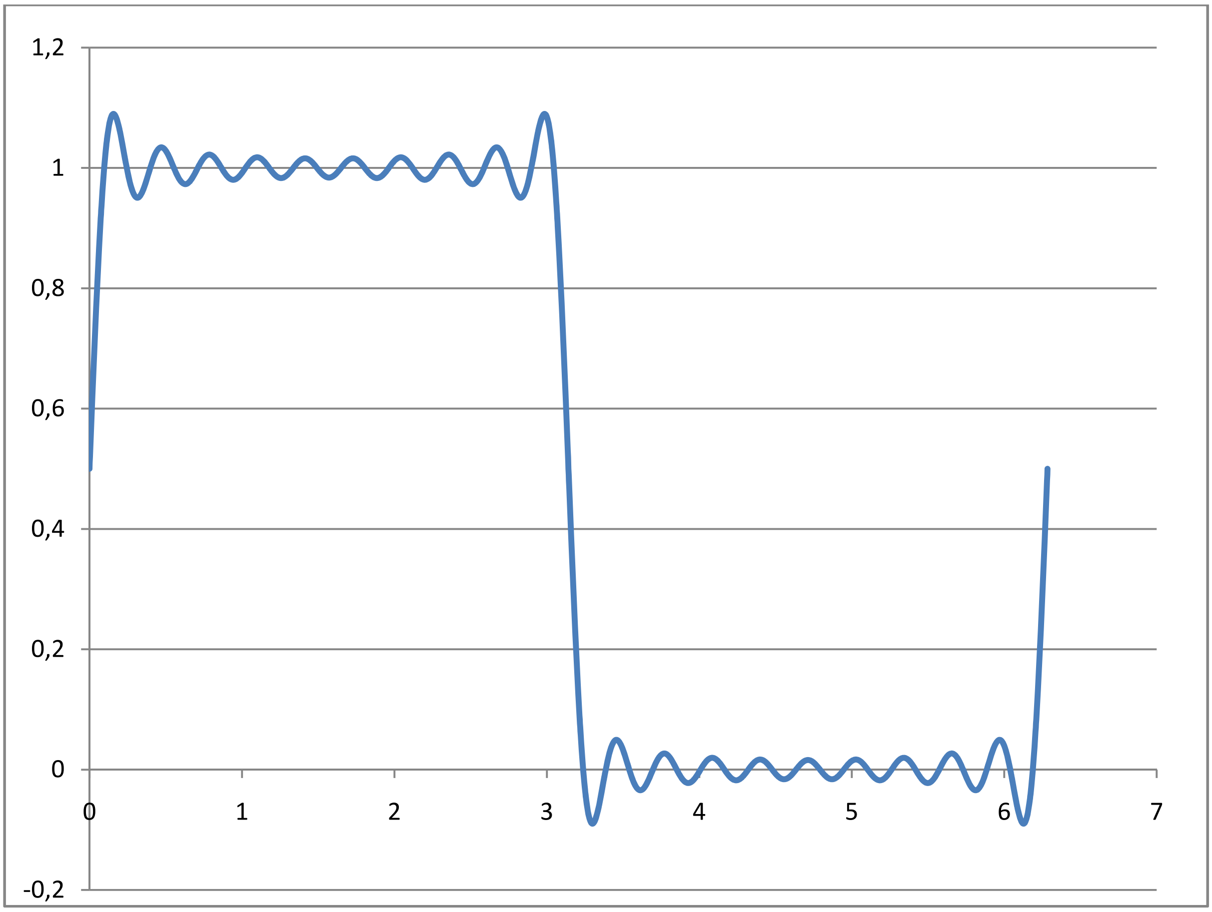
$$g(t) = c_0 + a_n.\cos(n.\omega.t) + b_n.\sin(n.\omega.t)$$

Grafy:

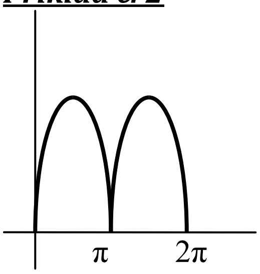
Harmonické složky:



Výsledný průběh:



Příklad č. 2



$s(t) = \sin(t)$ pro $t \in \langle 0; \pi \rangle$

$\omega = 2$

$$c_0 = \frac{1}{\pi}.\int_0^{\pi} \sin(t)\,.\,dt = \frac{1}{\pi}.\left[\frac{-\cos(\omega.t)}{\omega}\right]_0^{\pi} = \frac{1}{\pi.\omega}.\{-\cos(\omega.\pi) + \cos(\omega.0)\} =$$

$$= \frac{1}{\pi.\omega}.\{1+1\} = \frac{2}{\pi.\omega}$$

$$c_n=\frac{1}{\pi}.\int_0^{\pi}\sin(\omega.t).e^{-j.n.\omega.t}.dt=\frac{1}{\pi}.\int_0^{\pi}\frac{e^{j.t}-e^{-j.t}}{2j}.e^{-j.n.\omega.t}.dt=$$

$$=\frac{1}{j.2.\pi}.\int_0^{\pi}(e^{j.t-j.n.\omega.t}-e^{-j.t-j.n.\omega.t}).dt=$$

$$=\frac{1}{j.2.\pi}.\int_0^{\pi}\left(e^{(j-j.n.\omega).t}-e^{(-j-j.n.\omega).t}\right).dt=$$

$$=\frac{1}{j.2.\pi}.\left[\frac{e^{(j-j.n.\omega).t}}{j-j.n.\omega}-\frac{e^{(-j-j.n.\omega).t}}{-j-j.n.\omega}\right]_0^{\pi}=$$

$$=\frac{1}{j.2.\pi}.\left[\frac{e^{j.((1-n.\omega).\pi)}-e^{(j-j.n.\omega).0}}{j.(1-n.\omega)}-\frac{e^{j.((-1-n.\omega).\pi)}-e^{(-j-j.n.\omega).0}}{j.(-1-n.\omega)}\right]=$$

$$=\frac{1}{j.2\pi}.\left[-\frac{\cos\left((1-n.\omega).\pi\right)+\overbrace{j.\sin\left((1-\mathrm{n}.\omega).\pi\right)}^{=0}-1}{1-n.\omega}.j+\right.$$

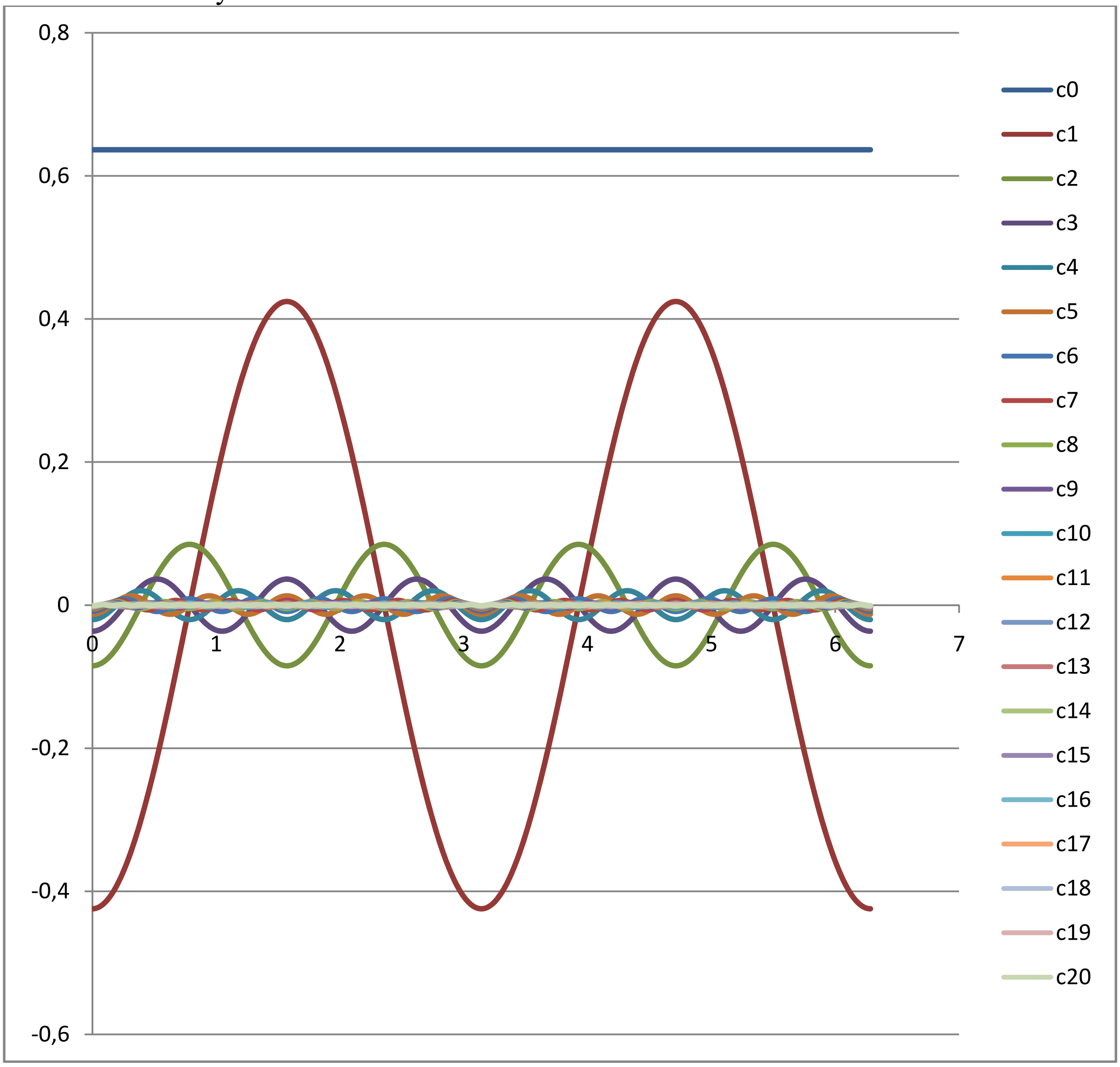
$$\left.+\frac{\cos\left((1+n.\omega).\pi\right)-\overbrace{j.\sin\left((1+n.\omega).\pi\right)}^{=0}-1}{-(1+n.\omega)}.j\right]=$$

$$=\frac{j}{j.2.\pi}.\left[-\frac{\overbrace{\cos\left((1-2.n).\pi\right)}^{=-1}-1}{1-2.n}-\frac{\overbrace{\cos\left((1+2.n).\pi\right)}^{=-1}-1}{1+2.n}\right]=$$

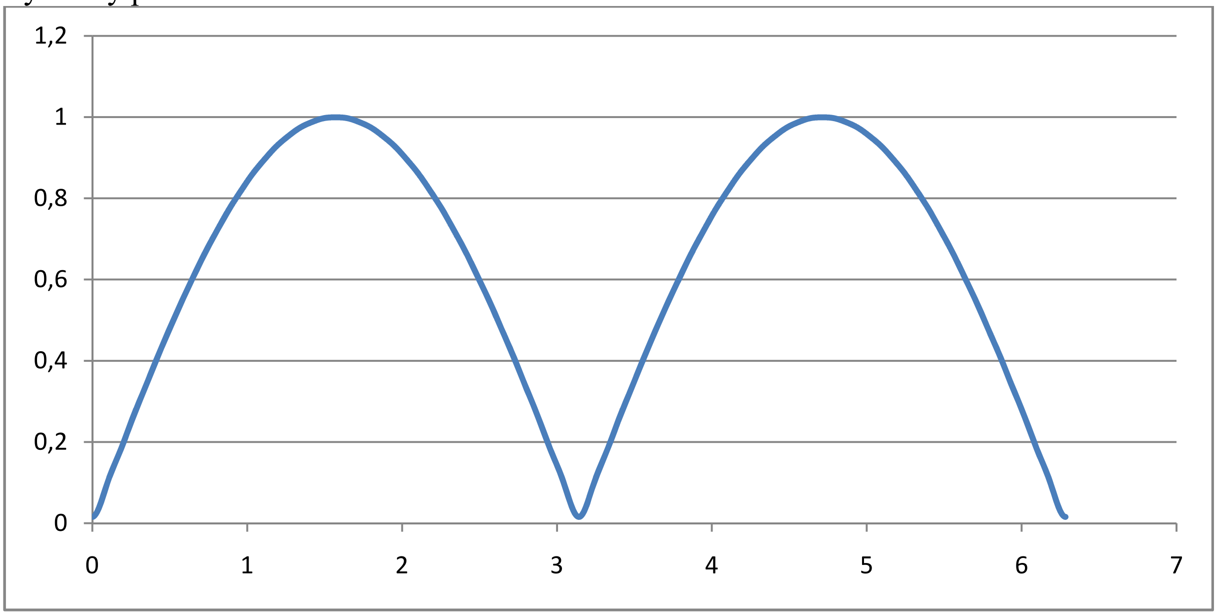
$$=\frac{1}{2.\pi}.\left[\frac{2}{1-2.n}+\frac{2}{1+2.n}\right]$$

Grafy:

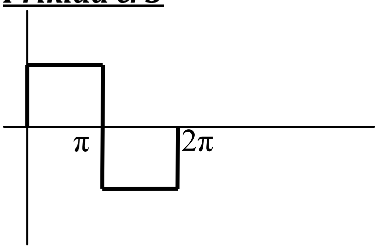
Harmonické složky:



Výsledný průběh:



Příklad č. 3



$\omega = 1$

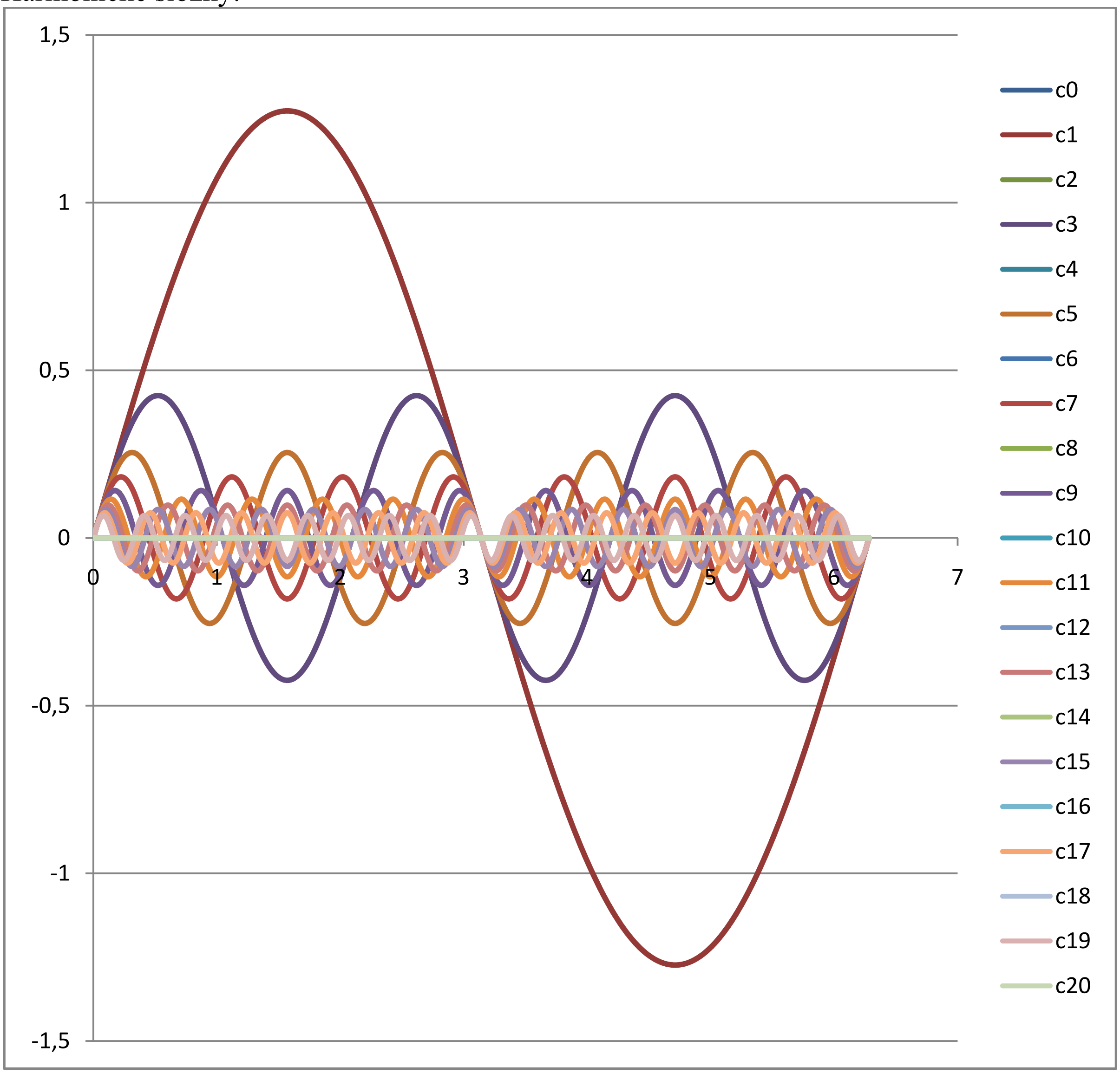
$$s(t) \begin{matrix} = 1 \text{ pro } t \in \langle 0; \pi \rangle \\ = -1 \text{ pro } t \in \langle \pi; 2\pi \rangle \end{matrix}$$

$$c_0 = \frac{1}{T} . \int_0^T s(t) . dt = \frac{1}{T} . \int_0^T (1 - 1) . dt = 0$$

$$
\begin{aligned}
c_n &= \frac{1}{T}.\int_0^T s(t).e^{-j.n.\omega.t}.dt = \frac{1}{T}.\left[\int_0^{\pi} e^{-j.n.\omega.t}.dt + \int_{\pi}^{2\pi} -1.e^{-j.n.\omega.t}.dt\right] = \\
&= \frac{1}{T}.\left\{\left[\frac{e^{-j.n.\omega.t}}{-j.n.\omega.t}\right]_0^{\pi} - \left[\frac{e^{-j.n.\omega.t}}{-j.n.\omega}\right]_{\pi}^{2\pi}\right\} = \\
&= \frac{1}{-j.n.\omega.T}.\left\{e^{-j.n.\omega.\pi} - 1 - e^{-j.n.\omega.2\pi} + e^{-j.n.\omega.\pi}\right\} = \\
&= \frac{1}{-j.n.\omega.T}.\Big\{\cos(-n.\omega.\pi) + \overbrace{j.\sin(-n.\omega.\pi)}^{=0} - 1 + \cos(-n.\omega.\pi) + \\
&+ \overbrace{j.\sin(-n.\omega.\pi)}^{=0} - \overbrace{\cos(-n.\omega.2\pi)}^{=1} - \overbrace{j.\sin(-n.\omega.\pi)}^{=0}\Big\} = \\
&= \frac{2}{-j.n.\omega.2\pi}.[\cos(-n.\omega.\pi) - 1] = \frac{1}{-j.n.\omega.\pi}.[\cos(n.\omega.\pi) - 1]
\end{aligned}
$$

Grafy:

Harmonické složky:



Výsledný průběh: