Fakulta elektrotechniky a informatiky
Univerzita Pardubice

Výpočty obvodů s obecným průběhem napětí

Semestrální práce z předmětu Lineární elektrické obvody

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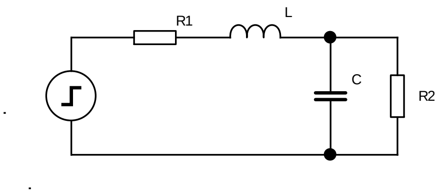
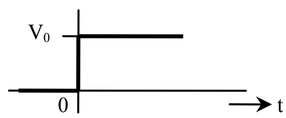
Zadání

V zadaném zapojení vypočtěte všechny veličiny: $i_{R1}(t)$, $i_L(t)$, $i_C(t)$, $i_{R2}(t)$, $u_{R1}(t)$, $u_L(t)$, $u_C(t)$, $u_{R2}(t)$.



Popis vstupního napětí:



Výpočty

Soupis všech rovnic:

$$i_{R1}(t) = i_C(t) + i_{R2}(t)$$

$$v_{R1}(t) = R1.i_{R1}(t)$$

$$v_L(t) = L.\frac{di_L(t)}{dt}$$

$$i_L(t) = i_{R1}(t)$$

$$v_C(t) = v_C(t_0) + \frac{1}{C}.\int i_C(t).dt$$

$$i_C(t) = i_{R1}(t) - i_{R2}(t)$$

$$i_{R2}(t) = \frac{v_C(t)}{R2}$$

$$v(t) = v_{R1}(t) + v_L(t) + v_C(t)$$

$$v(t) = V_0$$

Výpočet proudu kondenzátoru $i_C(t)$:

$$v(t) = i_R(t).R1 + L.\frac{di_R(t)}{dt} + v_C(t_0) + \frac{1}{C}.\int i_C(t).dt$$

$$v(t) = R1.\left[i_C(t) + \frac{1}{C.R2}.\int i_C(t).dt\right] + L.\frac{d\left[i_C(t) + \frac{1}{C.R2}.\int i_C(t).dt\right]}{dt} + v_C(t_0) + \frac{1}{C}.\int i_C(t).dt$$

$$v(t) = R1.i_C(t) + \frac{R1}{C.R2}.\int i_C(t).dt + L.\frac{di_C(t)}{dt} + \frac{L}{C.R2}.i_C(t) + v_C(t_0) + \frac{1}{C}.\int i_C(t).dt \quad \Big| \frac{d}{dt}$$

$$0 = R1.\frac{di_C(t)}{dt} + \frac{R1}{C.R2}.i_C(t) + L.\frac{d^2 i_C(t)}{dt} + \frac{L}{C.R2}.\frac{di_C(t)}{dt} + \frac{1}{C}.i_C(t)$$

$$L.\frac{d^2 i_C(t)}{dt} + \left[R1 + \frac{L}{C.R2}\right].\frac{di_C(t)}{dt} + \left[\frac{R1}{C.R2} + \frac{1}{C}\right].i_C(t) = 0$$

$$i_C(t) = k.e^{\lambda t} \Rightarrow \frac{di_C(t)}{dt} = k.\lambda.e^{\lambda t} \Rightarrow \frac{d^2 i_C(t)}{dt} = k.\lambda^2.e^{\lambda t}$$

$$L.k.\lambda^2.e^{\lambda t} + \left[R1 + \frac{L}{C.R2}\right].k.\lambda.e^{\lambda t} + \left[\frac{R1}{C.R2} + \frac{1}{C}\right].k.e^{\lambda t} = 0 \quad |: k.e^{\lambda t}$$

$$L.\lambda^2 + \left[R1 + \frac{L}{C.R2}\right].\lambda + \left[\frac{R1}{C.R2} + \frac{1}{C}\right] = 0$$

$$D=\left(R1+\frac{L}{C.R2}\right)^2-4.L.\left(\frac{R1}{C.R2}+\frac{1}{C}\right)=R1^2+\frac{2.R1.L}{C.R2}+\frac{L^2}{C^2.R2^2}-\frac{4.L.R1}{C.R2}-\frac{4.L}{C}=$$

$$=R1^2-2.\frac{R1.L}{C.R2}+\frac{L^2}{C^2.R2^2}-4.\frac{L}{C}$$

$$\lambda_{1,2}=\frac{-\left(R1+\frac{L}{C.R2}\right)\pm j.\sqrt{R1^2-2.\frac{R1.L}{C.R2}+\frac{L^2}{C^2.R2^2}-4.\frac{L}{C}}}{2L}=$$

$$=\underbrace{\frac{-\left(R1+\frac{L}{C.R2}\right)}{2L}}_{\alpha}\pm j.\underbrace{\frac{\sqrt{R1^2-2.\frac{R1.L}{C.R2}+\frac{L^2}{C^2.R2^2}-4.\frac{L}{C}}}{2L}}_{\beta}$$

Obecný vztah pro $i_C(t)$:

$$i_C(t)=k1.e^{\alpha t}.\sin(\beta t)+k2.e^{\alpha t}.\cos(\beta t)$$

Výpočet konstant $k1$ a $k2$:

$$i_C(t=0)=k1.e^{\alpha t}.\sin(\beta t)+k2.e^{\alpha t}.\cos(\beta t)=0\Rightarrow k2=0$$

$$i_C'(t=0)=k1.\alpha.e^{\alpha t}.\sin(\beta t)+k1.e^{\alpha t}.\cos(\beta t).\beta+k2.\alpha.e^{\alpha t}.\cos(\beta t)-k2.e^{\alpha t}.\sin(\beta t).\beta=$$

$$=\frac{V_0}{L}\Rightarrow k1=\frac{V_0}{L.\beta}$$

Výsledný vztah pro $i_C(t)$:

$$i_C(t)=\frac{V_0}{L.\beta}.e^{\alpha t}.\sin(\beta t)$$

Výpočet napětí na kondenzátoru $u_C(t)$:

$$u_C(t)=\frac{1}{\mathrm{C}}.\int_0^t i_C(\tau).d\tau=\frac{1}{\mathrm{C}}.\int_0^t\frac{V_0}{L.\beta}.e^{\alpha\tau}.\sin(\beta\tau).d\tau=\frac{V_0}{C.L.\beta}.\int_0^t e^{\alpha\tau}.\sin(\beta\tau).d\tau$$

$u'=\alpha.e^{\alpha\tau}$ $\quad v=-\frac{1}{\beta}.\cos(\beta\tau)$ $\qquad\qquad u'=\alpha.e^{\alpha\tau}$ $\quad v=\frac{1}{\beta}.\sin(\beta\tau)$

$$\int_0^t\overbrace{e^{\alpha\tau}}^{u}.\overbrace{\sin(\beta\tau)}^{v'}.d\tau=-\frac{1}{\beta}.[e^{\alpha\tau}.\cos(\beta\tau)]_0^t+\frac{\alpha}{\beta}.\int_0^t\overbrace{e^{\alpha\tau}}^{u}.\overbrace{\cos(\beta\tau)}^{v'}.d\tau=$$

$$=-\frac{1}{\beta}.[e^{\alpha t}.\cos(\beta t)-1]+\frac{\alpha}{\beta}.\left\{\frac{1}{\beta}.[e^{\alpha\tau}.\sin(\beta\tau)]_0^t-\frac{\alpha}{\beta}.\int_0^t e^{\alpha\tau}.\sin(\beta\tau).d\tau\right\}=$$

$$=-\frac{1}{\beta}.[e^{\alpha t}.\cos(\beta t)-1]+\frac{\alpha}{\beta}.\left\{\frac{1}{\beta}.e^{\alpha t}.\sin(\beta t)-\frac{\alpha}{\beta}.\int_0^t e^{\alpha\tau}.\sin(\beta\tau).d\tau\right\}$$

- Výpočet integrálu – pomocí substituce

$$\int_0^t e^{\alpha\tau}.\sin(\beta\tau).d\tau=I=-\frac{1}{\beta}.[e^{\alpha t}.\cos(\beta t)-1]+\frac{\alpha}{\beta^2}.e^{\alpha t}.\sin(\beta t)-\frac{\alpha^2}{\beta^2}.I$$

$$I=\frac{-\frac{1}{\beta}.[e^{\alpha t}.\cos(\beta t)-1]+\frac{\alpha}{\beta^2}.e^{\alpha t}.\sin(\beta t)}{1+\frac{\alpha^2}{\beta^2}}=\int_0^t e^{\alpha\tau}.\sin(\beta\tau).d\tau$$

Výsledný vztah pro napětí na kondenzátoru $u_C(t)$:

$$u_C(t)=\frac{V_0}{L.C.\beta}.\frac{-\frac{1}{\beta}.[e^{\alpha t}.\cos(\beta t)-1]+\frac{\alpha}{\beta^2}.e^{\alpha t}.\sin(\beta t)}{1+\frac{\alpha^2}{\beta^2}}$$

Výpočet a výsledný vztah pro proud odporem R2 $i_{R2}(t)$:

$$i_{R2}(t)=\frac{u_C(t)}{R2}=\frac{V_0}{R2.L.C.\beta}.\frac{-\frac{1}{\beta}.[e^{\alpha t}.\cos(\beta t)-1]+\frac{\alpha}{\beta^2}.e^{\alpha t}.\sin(\beta t)}{1+\frac{\alpha^2}{\beta^2}}$$

Výpočet a výsledný vztah pro proud odporem R1 $i_{R1}(t)$:

$i_{R1}(t)=i_L(t)=i_C(t)+i_{R2}(t)=$

$$=\frac{V_0}{L.\beta}.e^{\alpha t}.\sin(\beta t)+\frac{V_0}{R2.L.C.\beta}.\frac{-\frac{1}{\beta}.[e^{\alpha t}.\cos(\beta t)-1]+\frac{\alpha}{\beta^2}.e^{\alpha t}.\sin(\beta t)}{1+\frac{\alpha^2}{\beta^2}}=$$

$$=\frac{V_0}{L.\beta}.\left\{e^{\alpha t}.\sin(\beta t)+\frac{1}{R2.C}.\frac{-\frac{1}{\beta}.[e^{\alpha t}.\cos(\beta t)-1]+\frac{\alpha}{\beta^2}.e^{\alpha t}.\sin(\beta t)}{1+\frac{\alpha^2}{\beta^2}}\right\}$$

Výpočet a výsledný vztah pro napětí na cívce $u_L(t)$:

$$u_L(t) = L.\frac{di_L(t)}{dt} =$$

$$= L.\frac{V_0}{L.\beta}.\left\{\frac{d[e^{\alpha t}.\sin(\beta t)]}{dt} + \right.$$

$$\left. + \frac{\frac{1}{R2.C}}{1+\frac{\alpha^2}{\beta^2}}.\left\{-\frac{1}{\beta}.\frac{d[e^{\alpha t}.\cos(\beta t)-1]}{dt} + \frac{\alpha}{\beta^2}.\frac{d[e^{\alpha t}.\sin(\beta t)]}{dt}\right\}\right\} =$$

$$= \frac{V_0}{\beta}.\left\{\alpha.e^{\alpha t}.\sin(\beta t) + e^{\alpha t}.\beta.\cos(\beta t) + \right.$$

$$+ \frac{\frac{1}{R2.C}}{1+\frac{\alpha^2}{\beta^2}}.\left\{-\frac{1}{\beta}.[\alpha.e^{\alpha t}.\cos(\beta t) - e^{\alpha t}.\beta.\sin(\beta t)] + \right.$$

$$\left.\left. + \frac{\alpha}{\beta^2}.[\alpha.e^{\alpha t}.\sin(\beta t) + e^{\alpha t}.\beta.\cos(\beta t)]\right\}\right\} =$$

$$= \frac{V_0}{\beta}.\left\{\alpha.e^{\alpha t}.\sin(\beta t) + e^{\alpha t}.\beta.\cos(\beta t) + \right.$$

$$+ \frac{\frac{1}{R2.C}}{1+\frac{\alpha^2}{\beta^2}}.\left\{-\frac{\alpha}{\beta}.e^{\alpha t}.\cos(\beta t) + e^{\alpha t}.\sin(\beta t) + \frac{\alpha^2}{\beta^2}.e^{\alpha t}.\sin(\beta t) + \right.$$

$$\left.\left. + \frac{\alpha}{\beta}.e^{\alpha t}.\cos(\beta t)\right\}\right\} =$$

$$= \frac{V_0.e^{\alpha t}}{\beta}\left\{\alpha.\sin(\beta t) + \beta.\cos(\beta t) + \frac{1}{R2.C}.\frac{\sin(\beta t) + \frac{\alpha^2}{\beta^2}.\sin(\beta t)}{1+\frac{\alpha^2}{\beta^2}}\right\} =$$

$$= \frac{V_0.e^{\alpha t}}{\beta}.\left\{\alpha.\sin(\beta t) + \beta.\cos(\beta t) + \frac{1}{R2.C}.\frac{\sin(\beta t).\left(1+\frac{\alpha^2}{\beta^2}\right)}{1+\frac{\alpha^2}{\beta^2}}\right\} =$$

$$= \frac{V_0.e^{\alpha t}}{\beta}.\left\{\beta.\cos(\beta t) + \sin(\beta t).\left[\alpha + \frac{1}{R2.C}\right]\right\}$$

Výpočet a výsledný vztah pro napětí na odporu R1 $u_{R1}(t)$:

$$u_{R1}(t) = i_{R1}(t).R1 = \frac{R1.V_0}{L.\beta}.\left\{e^{\alpha t}.\sin(\beta t) + \frac{1}{R2}.\frac{-\frac{1}{\beta}.[e^{\alpha t}.\cos(\beta t) - 1] + \frac{\alpha}{\beta^2}.e^{\alpha t}.\sin(\beta t)}{1 + \frac{\alpha^2}{\beta^2}}\right\}$$

Grafy všech průběhů

Pro výpočty byly použity tyto součástky:

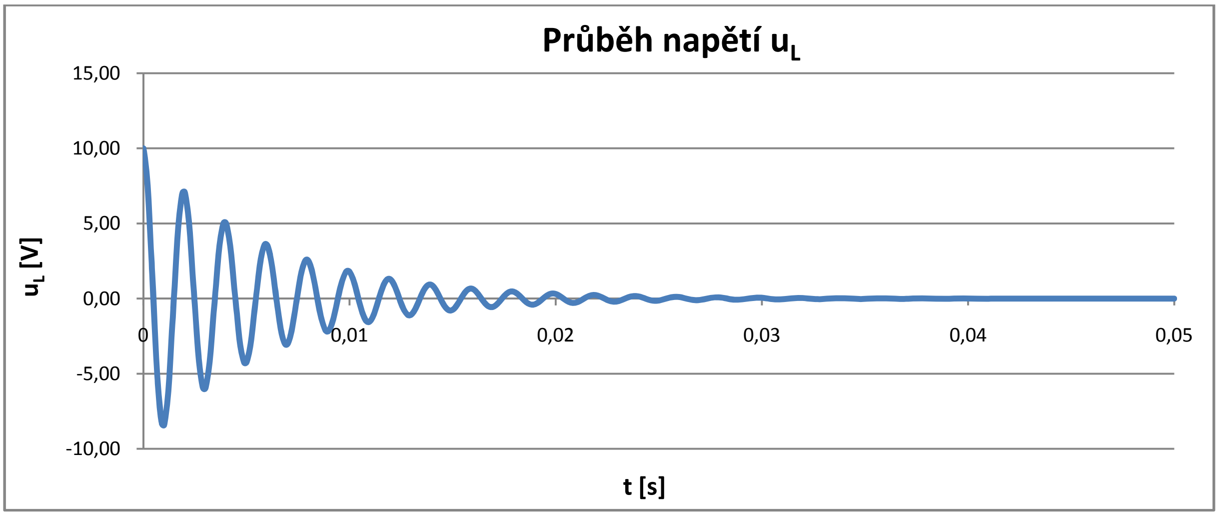
R1 = 33Ω

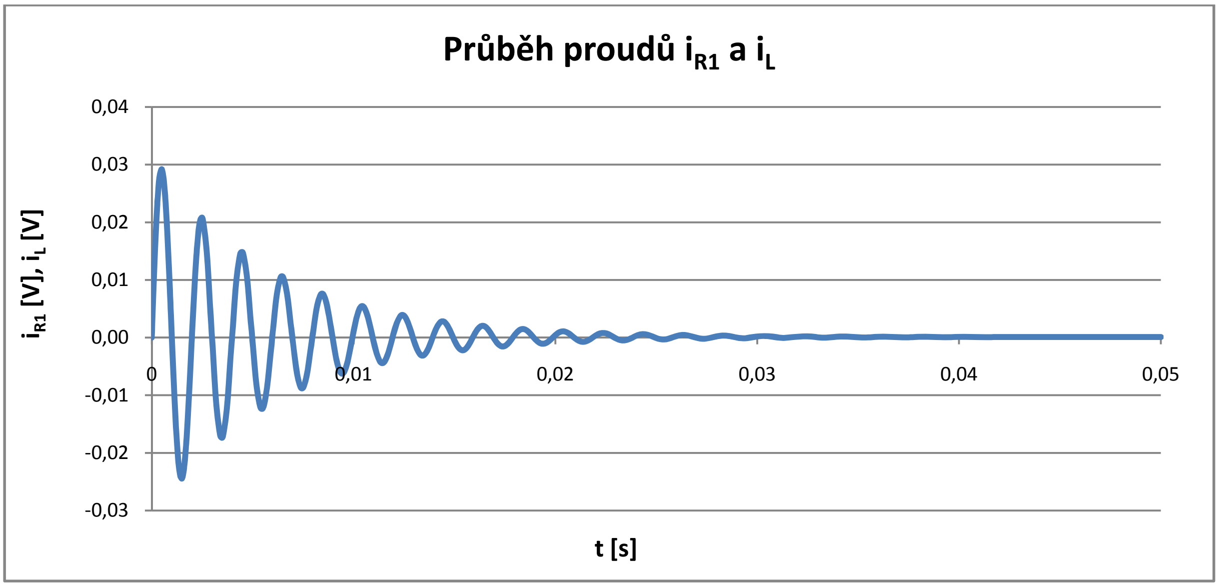
R2 = 100kΩ

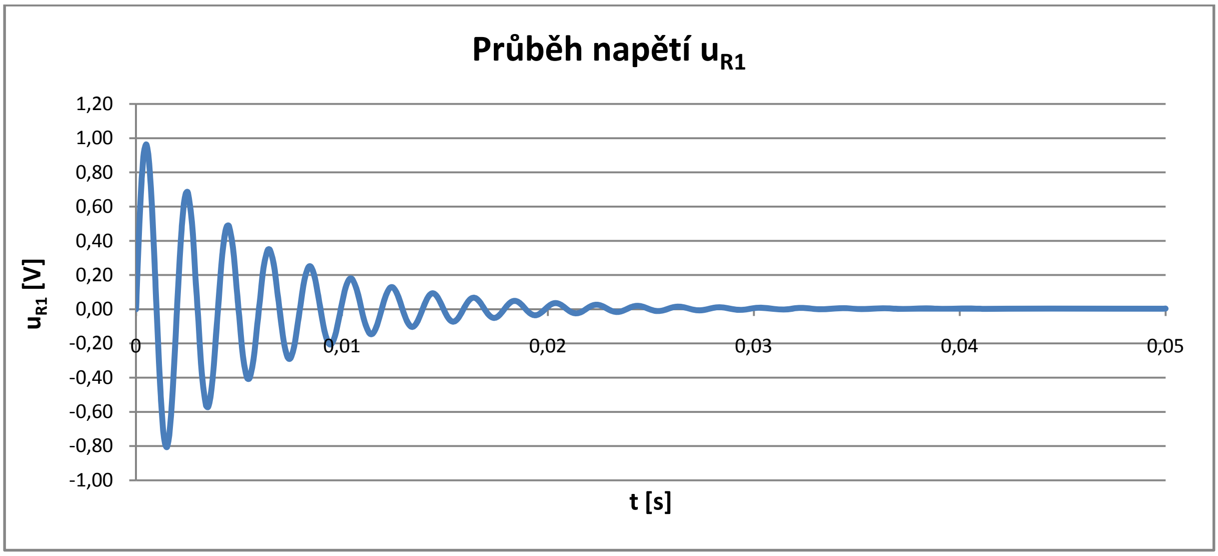
L = 100mH

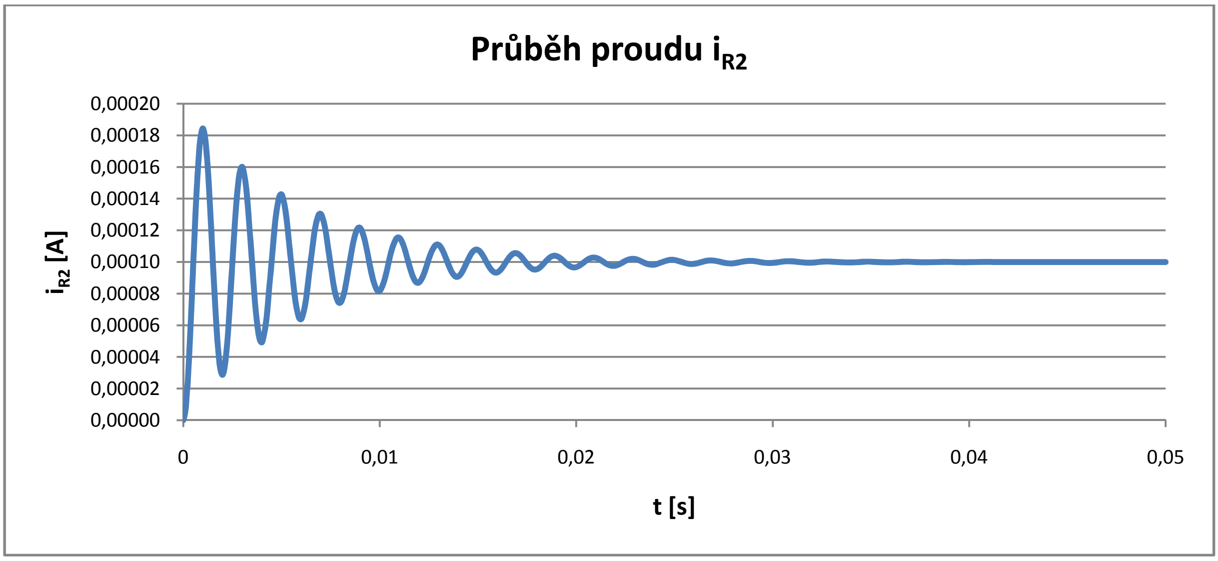
C = 1μF

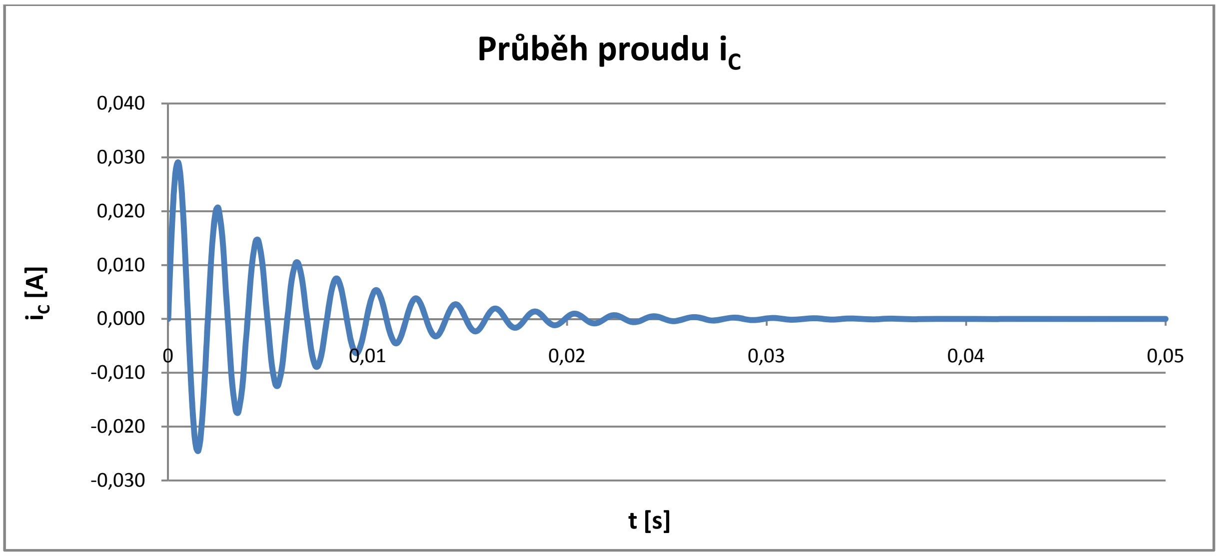
V_0 = 10V

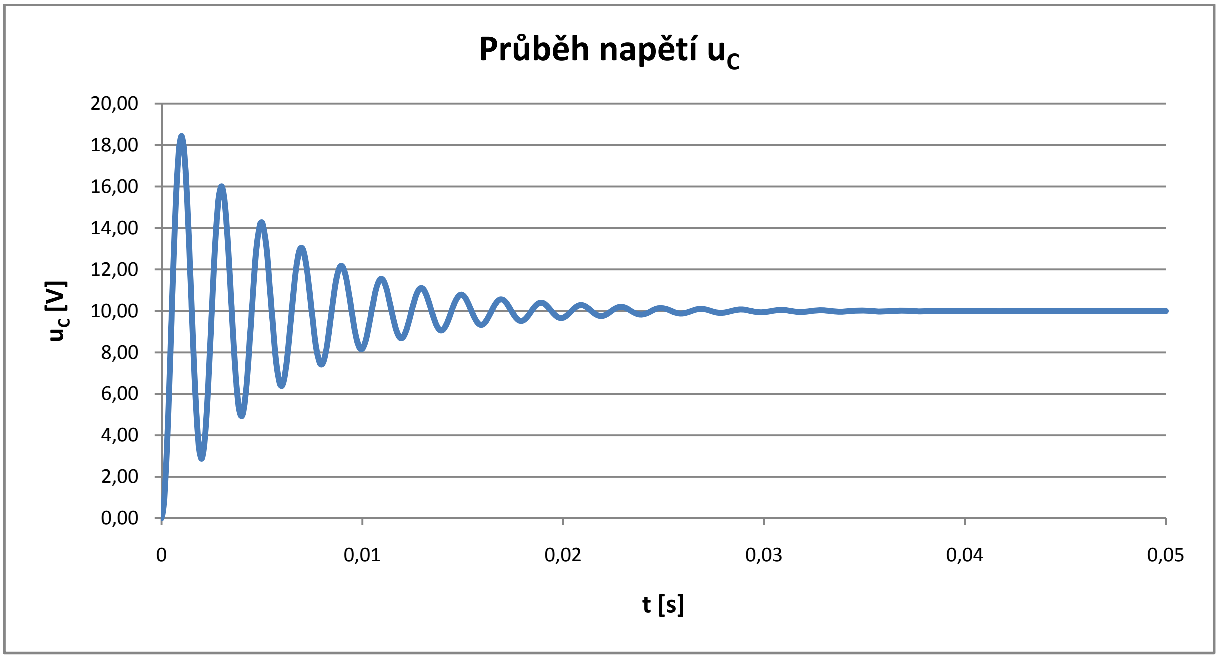





Průběh napětí u_{R1}
1,20
1,00
0,80
0,60
0,40
0,20
0,00
-0,20
-0,40
-0,60
-0,80
-1,00
u_{R1} [V]
0
0,01
0,02
0,03
0,04
0,05
t [s]


Průběh proudu i_{R2}
0,00020
0,00018
0,00016
0,00014
0,00012
0,00010
0,00008
0,00006
0,00004
0,00002
0,00000
i_{R2} [A]
0
0,01
0,02
0,03
0,04
0,05
t [s]


Průběh proudu i_C
0,040
0,030
0,020
0,010
0,000
-0,010
-0,020
-0,030
i_C [A]
0
0,01
0,02
0,03
0,04
0,05
t [s]



Pro kontrolu přikládám i průběh napětí $v(t)$, tedy součet všech napětí